NATHANSON

A Helical Method for the Determination of $\frac{e}{m}$ & v for Wehnelt Cathode Rays

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A HELICAL METHOD FOR THE DETERMINATION OF $\frac{e}{m}$ AND v FOR WEHNELT CATHODE RAYS

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

JONAS BERNARD NATHANSON

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Recommendation concurred in:

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CHAPTER I

Introduction

The discovery of Röntgen rays in 1896 by Röntgen, opened a new field of research in physics, and the beauty and remarkable properties of the cathode rays which are the source of the Röntgen rays, lent sufficient inspiration to the physicists during the last seventeen years so that we have as a result, the establishment of new theories of electricity, matter and magnetism. Some of the mysteries of electricity whose actions are so well known but whose nature lies so much shrouded in mystery, seem to be partly revealed when a conductor assumes the shape of a discharge tube permitting the investigator to examine the phenomena occuring within. Here seems to be the most promising point of attack for the investigation of the ultimate nature of matter. Indeed the problem has been attacked from theoretical and experimental standpoints resulting in the establishment of our present electron theory of electricity.

Investigation of the nature of cathode[1] rays brought out the fact that these rays consist of a stream of negatively charged particles. A determination of the important quantity $\frac{e}{m}$, i.e., the ratio of the charge to the mass of one of these negative particles or ions, brought forth the astounding fact that $\frac{e}{m}$ was of the order 10^7 electromagnetic units while $\frac{e}{m}$ for ions in electrolytes was known to be only of the order 10^4. The only deductions possible were either that the charge on an ion in gases was larger than the charge on an ion in electrolytes, or else that the mass of the gaseous ion was much smaller than the mass of the ion in electrolytes. For many good and valid

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[*]J.J. Thomson, Phil. Mag. V. 44, p. 293, 1897.

reasons e for a gaseous ion or for an electrolytic ion is the same, hence investigators have been led to the important conclusion, that the mass of the gaseous ion is about 1700 times as small as a H atom. This conclusion is startling since it thus appears that the ultimate unit of matter is not the atom but the gaseous ion or electron. The ratio $\frac{e}{m}$ is thus seen to revolutionize our old chemical notions of matter, and hence the true establishment of this ratio is as important a matter as the establishment of any other great constant of nature.

Furthermore, the electron theory has revolutionized our ordinary conception of mechanics. Newton's laws of motion which held unchallenged sway for 300 years, seem not to hold for bodies having a velocity approaching that of light. Experimental work by Kaufmann, Bucherer and others have verified the theories advanced by Lorentz, Einstein, Abraham and Bucherer, that the mass of a body increases with increasing velocities approaching that of light. Variation in the mass of an electron means a variation in the ratio $\frac{e}{m}$, and the establishment of a precise value for $\frac{e}{m}$ opens a way for the verification of the various theoretical formulae advanced.

We thus see the great importance of $\frac{e}{m}$ for negative carriers of electricity, in the theoretical physics of the present day. To test the constancy of this ratio (for velocities small compared to that of light) is a worthy task, for the more determinations made of this quantity, and the greater number and variety of the methods used, the more firmly is the constancy of this value determined.

CHAPTER II

The immense amount of work done on the discharge of electricity through gases and allied subjects, has disclosed the fact that there is more than one source of negative carriers of electricity outside of the ordinary cathode discharge. As a result $\frac{e}{m}$ has been determined for -

1 Cathode rays emitted in ordinary discharge tubes from cold cathodes.

2 Cathode rays that have passed through thin metal sheets, i.e., Lenard rays.

3 Cathode rays reflected from heavy metals like Cu.

4 Cathode rays having their origin in the impact of Röntgen rays on a metal like Pt.

5 Becquerel or β rays emitted from radio-active substances,- these rays being similar in nature to cathode rays.

6 Negative particles emitted by negatively charged metals under the influence of ultra-violet light.

7 Negative carriers emitted by negatively charged insulators under the influence of ultra-violet light.

8 Negative carriers emitted from incandescent metals and carbon

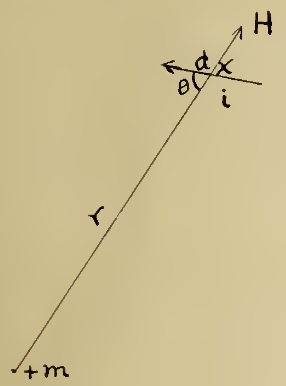
9 Negative carriers emitted from incandescent oxides, e.g., Wehnelt cathode rays.

In the work of this paper, the last was chosen as the source of negative carriers of electricity, as will be more fully explained later.

There are a few general principles employed in the determination of $\frac{e}{m}$ and v (velocity), that will be now presented, as they are frequently met with in different methods employed. In all the determinations of $\frac{e}{m}$ for negative carriers, the hypothesis of an emission

of negative particles has been assumed, e.g., cathode rays are considered as being a stream of negatively charged particles (each having a charge e) thrown off from the cathode with a high velocity. If this is the case then this stream of negative particles, or electrons as they are sometimes known, is equivalent to an electric current, for an electric current is nothing more than a stream of electric charges.

Evidently the faster these particles or electrons move, the greater will be the electric current. The effect of a magnetic field on this current will be an electro-magnetic deflection, this deflection being proportional to the magnitude of this current. We can thus see in a general way, that a knowledge of the amount of the magnetic deflection of a stream of electrons[2] will give us an index to the magnitude of their velocity.



From the elementary law of electromagnetic action, we know that the force exerted on an element of current idx by a magnetic pole of strength m, distant r from dx, is

$$dF = \frac{m\ idx\ \sin\theta}{r^2}$$

$$= H\ i\ dx\ \sin\theta$$

where

$$H = \text{magnetic force}$$

Therefore

$$dF = H\ i\ dt\ \frac{dx}{dt}\ \sin\theta$$

If

$$\sin\theta = 1$$

Therefore

$$dF = H\ idt\ \frac{dx}{dt}\ .$$

If the current is considered as being composed of a stream of particles each of mass m, then

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2 The terms negative particles and electrons will be used interchangeably in this paper.

$$dF = m\frac{d^2y}{dt^2} = He\frac{dx}{dt} = Hev \qquad (1)$$

where

y = direction of magnetic deflection

and

$e = idt$ = charge on a particle

v = velocity

Since electrons are charged particles, we should expect that a stream of cathode particles, for example, would be deflected by an electrostatic field. Indeed Hertz[3] in 1885 had failed to detect an electrostatic deflection, but this was because he did not work at a high enough vacuum, with the result that the gas between his electrostatic plates acted like a conductor hence shielding the cathode particles from the electrostatic plates. Since then, the electrostatic deflection of cathode particles has been fully established.

If X is the electric force acting on unit charge. Then upon a charge e, the force would be Xe. This imparts an acceleration in the z direction, for example, equal to $m\frac{d^2z}{dt^2}$ where m = mass of particle. Therefore

$$m\frac{d^2z}{dt^2} = Xe \ . \qquad (2)$$

Equations (1) and (2) will suffice to determine $\frac{e}{m}$ and v.

The problem might be attacked differently, i.e., from energy relationships. Let V = potential difference between anode and cathode of a discharge tube. Then the energy imparted to a charge e is Ve, and if all of this energy goes to impart kinetic energy to the particle then

$$Ve = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}mv^2 \qquad (3)$$

A combination of equations (1) and (3) will therefore serve to determine $\frac{e}{m}$ and v.

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3 Wied. Ann. 19, p. 782, 1883.

Another principle used has been the principle of equating heat given up by the sudden stopping of the particles, where all of the kinetic energy of the particles is converted into heat.

Briefly therefore $\frac{e}{m}$ has been, in general, determined from a measurement of magnetic and electrostatic deflections, and from a knowledge of the energy of the moving particles, as obtained from the potential difference of cathode and anode, or from heat given up on dissipation of the kinetic energy of the moving particles.

CHAPTER III

$\frac{e}{m}$ and v for Cathode and β Rays

Before proceeding to the work to be described in this paper, it might be well to take a general survey of the work done on $\frac{e}{m}$ and v for negative carriers of electricity, by previous workers. It will be attempted to present the work of the most noteworthy investigators in a brief and simple manner, giving only the general principles underlying various methods used, and avoiding unnecessary and burdensome details as much as possible.

A. Schuster[4] was the first to determine $\frac{e}{m}$ by means of the magnetic deflection of the cathode rays. The values that he determined for $\frac{e}{m}$ led him to the false conclusion, that the electric particles within a discharge tube are identical with ordinary atoms.

His method is as follows. Considering as on page 5, that the energy of the electric field is spent in imparting kinetic energy to the moving particles, we have

$$Ve = \frac{1}{2} mv^2 \qquad (1)$$

where

v = velocity of the particles.

If the particle moves through a magnetic field perpendicular to the lines of force, it will describe a circle, such that it is accelerated towards the center of the circle with a force Hev (page 5). This must be equal to the centrifugal force of the moving particle, or

$$Hev = \frac{mv^2}{r}$$

where

r = radius of the circle.

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4 Proc. Roy. Soc. 47, p. 526, 1890.

From equations (1) and (2) we have,

$$\frac{e}{m} = \frac{2V}{H^2r^2}$$

V, H and r being known.

In this work, it is assumed that all of the energy of the electric field is utilized in imparting kinetic energy to the particles. But this is somewhat doubtful for some of the energy might have been employed in extracting the particle from the cathode. Even Schuster himself has doubted the validity of equation (1), as he says, "The assumption that in the passage of the particles, the work done appears as acceleration, can never be perfectly realized, and experiments only can decide how nearly we may approach it".

It is evident that equation (1) will give us an upper limit for $\frac{e}{m}$, so that $\frac{e}{m} > 11 \times 10^5$ as Schuster determined.

To obtain a lower limit for $\frac{e}{m}$, he employed for the velocity v, values obtained from the kinetic theory of gases. Substituting in $\frac{e}{m} = \frac{v}{Hr}$, equation (2), he obtained $\frac{e}{m}$ about 10^3.

Hence $\frac{e}{m} < 11 \times 10^5$ and $> 10^3$.

Since this value lies about 10^4, the value for an H atom, he falsely concluded that the cathode particles are atomic in size, a conclusion repudiated by later workers.

Sir. J. J. Thomson[5] was the first to employ both electrostatic and magnetic deflections in the determination of $\frac{e}{m}$ for cathode rays. The lines of force of the two fields crossed each other at right angles so that the resulting magnetic and electrostatic deflections occured in the same direction. By balancing his two fields, Thomson was able to obtain no deflection. The equations representing

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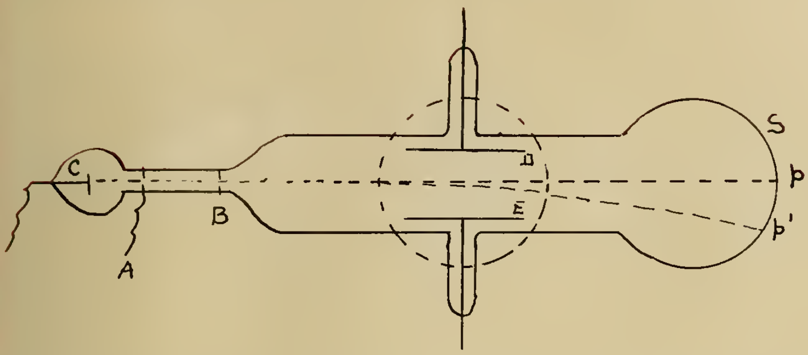
5 Phil. Mag., V, 44, p. 293, 1897.

this condition enabled him to determine both $\frac{e}{m}$ and v.



In the apparatus employed, the cathode beam issued from the cathode C and was reduced to a sharp pencil by passing first through the perforated anode A and then through the earthed screen B. The beam then traversed the crossed fields receiving an up or down deflection and then striking the phorphorescent screen S.

The electrostatic field was produced by the two charged plates D and E, the lines of force being up and down. The lines of magnetic force is perpendicular to the plane of this page.

With no fields, the rays passed through undeflected, striking the screen at p. Application of the magnetic field brought the spot down to p'. Application of an electrostatic field of proper strength brought the spot back to p.

The path of the particles under the influence of the magnetic field alone are circles, the centripetal and centrifugal forces being represented by the equation,

$$Hev = \frac{mv^2}{r}, \qquad (1)$$

where the letters have the same meaning as previously used. The radius r can be easily determined from the deflection pp', the distance between B and the screen, the undeflected beam being taken as tangent to the circle.

The force exerted on a particle by the electric field is Xe. When the two fields are balanced up to give no deflection we have the equation,

$$Xe = Hev$$

Therefore

$$v = \frac{X}{H}\ . \qquad (2)$$

From equations (1) and (2) $\frac{e}{m}$ and v can be determined. With this method Thomson obtained the values $\frac{e}{m} = 0.77 \times 10^7$ and $v = 2.5 \times 10^9$.

In the same year (1897) Thomson[6] determined $\frac{e}{m}$ and v by means of magnetic deflection, and by measuring the charge and heat given up by cathode rays on striking a thermo-couple. A narrow pencil of rays was allowed to fall on a thermo-couple whose rate of increase of temperature was measured. From this the amount of heat Q, communicated to the thermo-couple in unit time, became known. Assuming that all of the kinetic energy of the cathode rays is given up to the thermo-couple in the form of heat, we have,

$$\frac{1}{2} Nmv^2 = Q, \qquad (1)$$

where N = number of particles striking the couple in unit time. It is therefore evident that in this same time Ne units of electricity will be imparted to the thermo-couple. This charge of electricity, Ne, can be measured by receiving the beam in a Faraday cylinder and measuring the rate at which an electrometer connected to the cylinder charges

The magnetic deflection of the rays will give us the equation,

$$Hev = \frac{mv^2}{r}$$

or

$$\frac{e}{mv} = \frac{1}{Hr} \qquad (2)$$

From equation (1) we have

$$\frac{e}{mv^2} = \frac{Ne}{2Q}\ . \qquad (3)$$

Equations (2) and (3) suffice to give us $\frac{e}{m}$ and v. With this method

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6 Phil. Mag., V., 44, p. 302, 1897.

Thomson obtained a mean value of $\frac{e}{m} = 1.17 \times 10^7$, using air and H, and for v the value 2.7×10^9.

This method cannot be considered very accurate since it involves three measurements, i.e., magnetic field, electric charge, and heat. It is doubtful if the electrometers then used were of a very accurate pattern. Furthermore, the measurement of heat is always attended by large percentage errors. Then again, it is assumed that all of the kinetic energy is dissipated into heat, an assumption not entirely warranted. Another error involved is the leaking of the charge from the Faraday cylinder due to the gas rendered conducting by the cathode rays.

In 1898, P. Lenard[7] determined $\frac{e}{m}$ and v for Lenard rays, which are nothing more than cathode rays that have passed through a thin sheet of Al. Both magnetic and electric deflections were employed. After passing through the Al window the rays were narrowed to a thin beam by means of suitably earthed diaphragms, and then conducted between two electrostatic plates where after undergoing a deflection, they finally struck a phosphorescent screen.

The magnetic field had its direction parallel to the electrostatic lines of force, so that the deflections undergone by the beam due to both fields, were at right angles to each other. The deflection due to the electrostatic field is given by

$$S = \frac{e}{mv^2} Xcd \qquad (1)$$

where

d = distance of phosphorescent spot from center of condenser forming electric field

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7 Wied, Ann. 64, p. 279, 1898.

c = length of the plates in direction of ray.

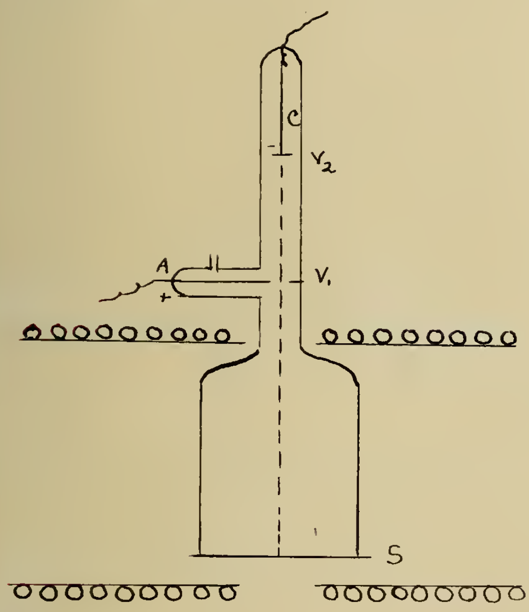
The deflection due to the magnetic field is given by

$$S = \frac{e}{mv} H c_1 d_1 \qquad (2)$$

where d_1 is the new distance of the phosphorescent spot from the center of the condenser, and c_1 = distance along which H is uniform.

Equations (1) and (2) suffice to determine $\frac{e}{m}$ and v. Lenard found the mean value for $\frac{e}{m} = 6.39 \times 10^6$ and $v = 0.73 \times 10^{10}$.

W. Kaufmann[8] obtained a much higher value for $\frac{e}{m}$ using Schuster's method, i.e., determining $\frac{e}{m}$ from a knowledge of potential difference between anode and cathode, and from magnetic deflections.



The cathode beam issuing from the cathode C passed through the perforated earthed anode A and then after traversing the uniform magnetic field produced a phosphorescent spot on S.

Let $\frac{dx}{dt}$ = velocity of the particles in direction of motion X.

Let $V_1 - V_2$ = difference of potential between anode and cathode.

The electric energy of the field being converted into kinetic energy of the particles, we have,

$$\frac{1}{2} m\left(\frac{dx}{dt}\right)^2 = e(V_1 - V_2). \qquad (1)$$

Again if y is the direction of the lines of magnetic force, and z is the direction of magnetic deflection, we have

$$m \frac{d^2z}{dt^2} = He \frac{dx}{dt} \qquad (2)$$

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8 Wied. ann., 61, p. 544, 1897.

But

$$\frac{d^2z}{dt^2} = \left(\frac{dx}{dt}\right)^2 \frac{d^2z}{dx^2}$$

Therefore

$$m\left(\frac{dx}{dt}\right)^2 \frac{d^2z}{dx^2} = He\,\frac{dx}{dt}\ . \qquad (3)$$

Substituting value of $\frac{dx}{dt}$ from (1) into (3),

$$\frac{d^2z}{dx^2} = H\sqrt{\frac{e}{2(V_1-V_2)m}} \qquad (4)$$

Integrating,

$$z = \frac{x^2}{2}\,H\,\sqrt{\frac{e}{m}\,\frac{1}{2(V_1-V_2)}} \qquad (5)$$

where $\frac{dz}{dx}$ and $z = 0$ when $x = 0$.

Kaufmann tested out equation (5) and found that for a constant field intensity H, the magnetic deflection z varied inversely with the square root of the potential difference between anode and cathode.

Equation (5) is true if the magnetic field is uniform for the path of the rays. If it is non-uniform the field integral $\int_0^{x_0} dx \int_0^x H dx$ must be used. Hence, we have

$$z = \sqrt{\frac{e}{2m(V_1-V_2)}}\ \int_0^{x_0} dx \int_0^x H dx\ . \qquad (6)$$

With equation (6) Kaufmann[9] obtained the value for $\frac{e}{m}$, 1.77×10^7.

In this work a few assumptions have been made that might be well to comment on. As has been previously said, we assumed above that all the electrical energy has been used to give kinetic energy to the moving particles, thus allowing no energy for extracting of the particle from the cathode. Furthermore some of the energy must have been lost in collisions with molecules of the rarified gas. Certainly in 1897, the high grade mercury pumps now in use, were then unknown. It is only at the highest vacuum, that the particles will actually start

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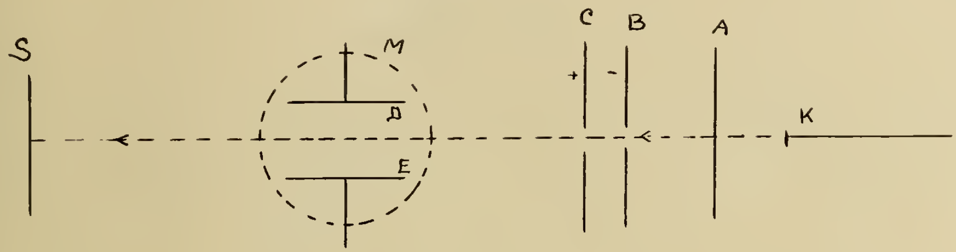
9 Wied. Ann., 62, p. 596, 1897.

on their path right from the surface of the cathode. This has of course also been assumed in Kaufmann's work.

Use of the method just described has been made by P. Lenard[10] working with Lenard rays which have a higher velocity than ordinary cathode rays. He however avoided the energy assumption in Kaufmann's work, by determining $\frac{e}{m}$ for rays that received added kinetic energy after having left the cathode.



The above is a diagrammatic arrangement of his apparatus. The beam issued from the cathode K, and passing through the thin Al sheet A, passed through an accelerating condenser BC. The latter consists of two plates oppositely charged. If C is positively charged and B negatively charged, then the negative particles after entering B with an initial velocity v_o will be repelled by B and attracted by C, so that they will receive an acceleration, leaving the condenser with a velocity v_1. The particles then pass between two electrostatic plates D and E and across a magnetic field M, finally striking the phosphorescent screen S. The increased kinetic energy imparted to the particles is thus made independent of the electrodes.

Let V_1 and V_2 be the potentials of the plates forming the

10 Wied. Ann., 65, p. 504, 1898.

accelerating condenser. Hence added kinetic energy imparted to the particles is,

$$(V_1 - V_2)e = \frac{1}{2} m(v_1 - v_0)^2 \qquad (1)$$

If the magnetic field strength is so adjusted that we have the same deflection for particles having v_0 and v_1, then

$$H_0 e v_0 = \frac{m v_0^2}{r}$$

and

$$H_1 e v_1 = \frac{m v_1^2}{r}$$

Therefore

$$\frac{H_0}{H_1} = \frac{v_0}{v_1} \qquad (2)$$

Equations (1) and (2) suffice to determine $\frac{e}{m}$, v_0 and v_1.

Lenard obtained a mean value of 6.8×10^6 for $\frac{e}{m}$, while for v_0 the values ranged from 0.62 to 0.88×10^{10} and for v_1 0.35 to 1.07×10^{10}.

S. Simon[11] in 1899 improved and refined Kaufmann's method, using an equation which was more rigorous than Kaufmann's. In this work Simon determined $\frac{e}{m}$ also from a knowledge of the potential difference of the electrodes, and from the magnetic deflection of the rays. The arrangement of his apparatus was essentially the same as Kaufmann's on page 12.

Consider the rays travleing along the x axis, the lines of magnetic force being in the y axis, so that the magnetic deflection is along the z axis. We have as previously,

$$\frac{He}{m} = \frac{v}{\rho}$$

where v = velocity of the rays in the x axis and ρ is the radius of curvature of the path of the rays under influence of the field.

Therefore

$$\frac{e}{m}\frac{H}{v} = \frac{1}{\rho} = \frac{\frac{d^2z}{dx^2}}{\left[1+\left(\frac{dz}{dx}\right)^2\right]^{\frac{3}{2}}}$$

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11 Wied. Ann., 69, p. 589, 1899.

Solving for $\frac{d^2z}{dx^2}$ and developing $\left[1 + \left(\frac{dz}{dx}\right)^2\right]^{\frac{3}{2}}$ by the binomial theorem, we have

$$\frac{d^2z}{dx^2} = \frac{e}{m}\frac{H}{v}\left[1 + \frac{3}{2}\left(\frac{dz}{dx}\right)^2 + \cdot\cdot\cdot\cdot\cdot\right] \qquad (1)$$

where the higher powers of $\frac{dz}{dx}$ are neglected, since the deflecti ons employed by Simon were very small. $\frac{dz}{dx}$ was evaluated from the r ight hand side of the equation by letting $\frac{dz}{dx} = 0$ and integrating $\frac{d^2z}{dx^2}$ which gave,

$$\frac{dz}{dx} = \frac{e}{mv}\int_0^x Hdx \qquad (2)$$

Substituting back this value for $\frac{dz}{dx}$ in the right hand side of equation (1), and integrating he finally obtained for $\frac{e}{m}$ the expression

$$\frac{e}{m} = \frac{2\,z_0'^2(V_1 - V_2)}{J^2\left[\int_0^{x_0} dx \int_0^x Hdx\right]^2} \qquad (3)$$

where

$$z_0'^2 = z_0^2\left\{1 - \frac{z_0^2 \int_0^x dx\,\left(\int_0^x Hdx\right)^3}{\left[\int_0^{x_0} dx \int_0^x Hdx\right]^3}\right\}$$

while the Kaufmann formula was (page 13, eq. 6) simply equation (3) with z_0 instead of the more accurate value z_0'.

Simon's value for $\frac{e}{m}$ was 1.865×10^7. In the above equation J = current strength, and v was evaluated by the equation

$$v = \sqrt{\frac{2e}{m}(V_1 - V_2)}$$

as on page 5, equation 3.

It will be noticed that Kaufmann's and Simon's values for $\frac{e}{m}$ are much higher than Thomson's or Lenard's values. This must be due to the different methods employed. However in all these methods the quantity $\frac{mv^2}{2e}$ is always involved. In Simon's work which seemed most accurate by virtue of the rigid equations used, we find the assumption that $\frac{mv^2}{2e} = V$, the difference of potential of the electrodes. In

Thomson's work using the thermo-couple, we find again $\frac{mv^2}{2e} = \frac{Q}{Ne}$, while in Lenard's work $\frac{mv^2}{2e}$ is also involved as a function of the potential.

W. Seitz[12] tested out the value $\frac{mv^2}{2e}$ for the three different methods, using the same discharge tube for all the different determinations. A bundle of cathode rays was conducted through suitably earthed diaphragms, and after passing between two electrostatic plates, the rays struck partly on a phosphorescent screen of uranium glass and partly on a bolometer. The vacuum employed was very high to avoid conduction of heat from the bolometer.

The quantity $\frac{Q}{Ne}$ was determined by measurement of the heat and charge given up to the bolometer. Now $\frac{Q}{Ne} = \frac{mv^2}{2e} = V$ the potential difference of the electrodes.. He found a very close agreement between $\frac{Q}{Ne}$ and V, barring of course the slight errors involved in the bolometric measurements.

To test $\frac{mv^2}{2e}$ for the electrostatic measurements as employed by Lenard page 11, he employed the equation of motion for the particles, i.e., $\frac{d^2y}{dt^2} = \frac{e}{m}\frac{\partial P}{\partial y}$, where P is the potential difference between the electrostatic plates. Integrating above expression we have,

$$y = \frac{e}{mv^2}\int_0^{x_0} dx \int_0^{x} \frac{\partial P}{\partial y}\, dx$$

where y is the electrostatic deflection. The integral was evaluated by graphical methods giving $\frac{mv^2}{2e} = KP$ where K is a constant. Again he found $\frac{mv^2}{2e}$ to agree very well with V the potential difference between anode and cathode.

Seitz also determined $\frac{e}{m}$ and v, using a magnetic field and applying Simon's equations to his apparatus, and evaluating the integrals graphically. His value of $\frac{e}{m}$ agreed very close with that of Simon's,

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12 Ann. d. phys., 8, p. 233, 1902.

having obtained for $\frac{e}{m}$ the value 1.87×10^7, and for v values ranging from .057 to 0.75×10^{10}, while Simon's value for $\frac{e}{m}$ was 1.865. The values for v cannot be compared so readily since v may vary with different investigators for different conditions employed. The order of v however is always about the same.

The identity in nature of Becquerel or β rays from radium, and cathode rays induced W. Kaufmann[13] to determine $\frac{e}{m}$ for β rays using a photographic method. The rays were emitted from a point source of radium bromide, and were deflected both by electrostatic and magnetic fields. The lines of force of the two fields coincided in direction so that the resulting magnetic and electrostatic deflections were at right angles to each other. After being deflected the rays struck a photographic plate. Since the radium emitted rays of different velocities, the result was a parabolic curve on the plate, to each point on the curve corresponding a definite value of $\frac{e}{m}$ and v.

The dimensions[14] of the apparatus used were small, since the intensity of the rays weakens for long paths, the distance from the source to the plate being only 2 cm. A high vacuum was also employed so as to avoid conduction between the electrostatic plates which were very close together. The whole apparatus was set in a uniform field.

The electrostatic deflection is given by the equation,

$$y = \frac{e}{m} \frac{X}{v^2} S_1 S_2 \qquad (1)$$

where S_1 and S_2 are constants of the apparatus and of the curve.

From the magnetic deflection we have,

$$\frac{e}{m} = \frac{v}{rH} \qquad (2)$$

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13 Nach. von d. Kgl. Gesell. zu Göttingen, Nov. 8, 1901.

14 Phys. Zeit., 2, p. 602, 1901.

where r is determined as a function of z the magnetic deflection.

The values obtainod for $\frac{e}{m}$ for values of v ranging from 2.36 to 2.83×10^{10} was $0.63 \times 1.31 \times 10^7$. These values when calculated back for v = 0 by theoretical formulae, gave

$$\frac{e}{m_o} = 1.95 \times 10^7.$$

In 1906 W. Kaufmann[15] investigated the constitution of the electron, using his previous photographic methods, but brought to a higher state of perfection. He used β rays again. The magnetic deflection z is given by

$$z = \frac{e}{mv} M \tag{1}$$

where M is the magnetic field integral.

The electrostatic deflection is given by

$$y = \frac{e}{mv^2} E \tag{2}$$

where E is the electric field integral.

Supplying experimental values for $\frac{e}{m}$ in the theoretical formulae of Abraham, Lorentz, and Bucherer for the variation of the mass of the electron with the velocity, he obtained for $\frac{e}{m_o}$, the velocity being zero, as follows,—

	$\frac{e}{m}$
After Abraham	1.823
After Lorentz	1.660
After Bucherer	1.808

Simon's value $\frac{e}{m} = 1.865$ when applied to the various formulae gave an average value of 1.878×10^7 for $\frac{e}{m}$ for v = 0.

Kaufmann's photographic method employing both electrostatic and magnetic deflections, has also been utilized by H. Starke[16] for

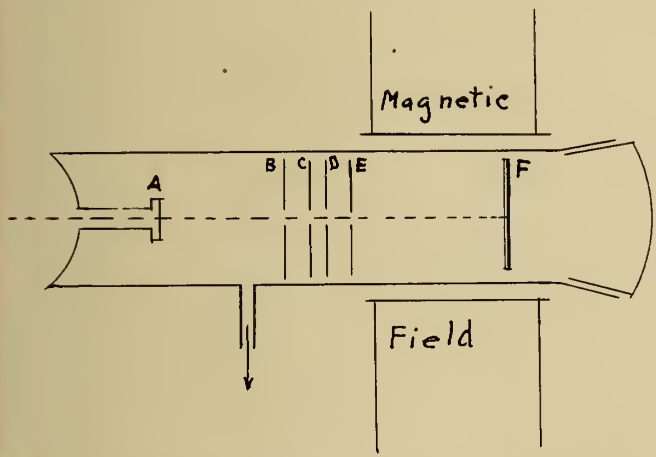
15 Ann. d. Phys., 19, p. 487, 1906.

16 Ver. d. Deut. Phys. Gesell., 5, p. 14, 1903.

cathode rays reflected from copper, as well as for the cathode rays passing through a thin sheet of metal. By suitable diaphragms a thin bundle of rays was separated from the mass of diffuse rays obtained by reflection and also by transmission through the thin Al sheet. The rays being somewhat heterogeneous he obtained for v values ranging from 3.66 to 5.64×10^9, for reflected rays, and for $\frac{e}{m}$, 1.84×10^7.

For rays passing through the Al sheet 0.002 mm. in thickness he obtained values for v ranging from 3.8 to 6.25×10^9, while for $\frac{e}{m}$ the value was 1.82×10^7.

August Becker[17] has made use of Lenard's acceleration for imparting kinetic energy to cathode particles, and Kaufmann's photographic methods for determining $\frac{e}{m}$ and v for cathode rays.



Cathode rays were conducted through a thin Al window A, then through a diaphragm B. The rays received an acceleration or retardation while passing through the condenser CD, depending upon the sign of the charges on CD. After going through another diaphragm E the rays were deflected in a magnetic field finally striking the photographic plate F. The vacuum was of the highest order, and the velocity of the particles was also high. The values of $\frac{e}{m}$ and v were obtained by the use of Simon's equations on page 16. The difference of potential used however, was that between the two plates of the accelerating condenser.

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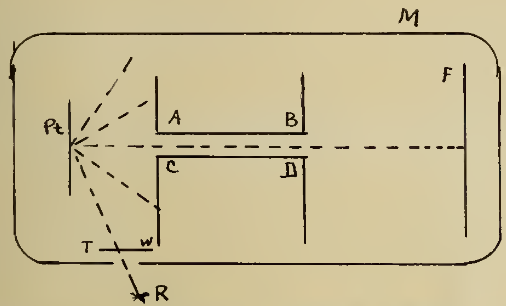
17 Ann. der Phys., 17, p. 381, 1905.

For a very high velocity, 1.11×10^{10}, Becker obtained the value $\frac{e}{m} = 1.747 \times 10^7$, while for a smaller velocity, $\frac{e}{m}$ was 1.847×10^7. This difference is of course due to the decrease of $\frac{e}{m}$ with increase of velocity.

The great success attending the work of investigators using the photographic method, led A. Bestolmeyer[18] in 1907 to use this method to determine $\frac{e}{m}$ for cathode rays that have been produced by the impact of Röntgen rays against a heavy metal like Pt.



Röntgen rays from an X-ray tube R, passed through the Al window TW and struck the Pt plate, from which were emitted cathode rays in all directions. The magnetic field due to the coil M, which surrounded the whole apparatus, and the electric field due to the condenser ABCD, acted in same direction but oppositely to each other, so that out of the great mass of heterogeneous rays arising from the Pt, there were singled out particles whose velocity was such, that those particles could pass through the condenser suffering no deflection. After passing through the condenser, whose plates were very close together, the rays suffered a magnetic deflection, finally striking the photographic plate F.

Since the particles were passing undeflected through the condenser, there must have been an equality between the forces exerted on the particles by the two fields, hence

$$Xe = Hev$$

or

$$v = \frac{X}{H} \qquad (1)$$

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18 Ann. der Phys., 22, p. 429, 1907.

After leaving the condenser only the magnetic field work on the particles, so that

$$r = \frac{mv}{eH} = \frac{m}{e}\frac{X}{H}, \qquad (2)$$

Equations (1) and (2) give $\frac{e}{m}$ and v, r being determined from the known dimensions of the apparatus and from the magnetic deflection. With this method Bestelmeyer obtained for $\frac{e}{m}$ the value 1.666×10^7 and for v, 0.818×10^{10}.

This value for $\frac{e}{m}$ Bestelmeyer considered small due to the high velocity of the particles. By means of the formulae of Abraham, Lorentz and Bucherer, he found that his value for $\frac{e}{m}$ would be about 1.72×10^7 when reduced to zero velocity.

This velocity is somewhat lower than tho Kaufmann-Simon value, i.e., 1.88×10^7. Bestelmeyer admitted his results might have been off 1 or 2% due to small inaccuracies in not having taken account of edge effects of his condenser, but it certainly was not off 8 or 9%, which was the amount his value differed from the Kaufmann-Simon value.

Bestelmeyer's method was improved upon by A. H. Bucherer[19] who determined $\frac{e}{m}$ for Becquerel rays which were emitted from a point source. Bucherer used a uniform magnetic field, having the magnetic coils cooled by water to a certain temperature. The electrostatic field consisted of two circular plates, and he investigated and corrected the errors introduced by the end effects of these plates. The vacuum was the highest obtainable using a Gaede pump.

The value obtained for $\frac{e}{m}$ was 1.763×10^7 correct to 0.5%. The values for v varied from 0.3173 to 0.6870 of the velocity of light.

The great care and accuracy employed by Bucherer, places the above value for $\frac{e}{m}$ as about the most accurate known.

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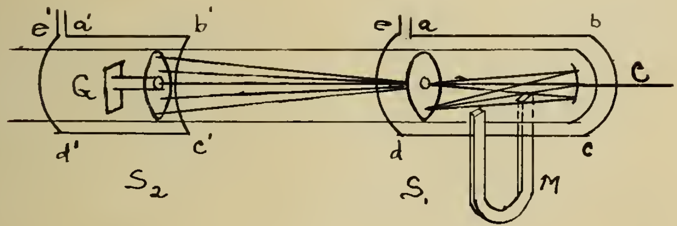
19 Ann. der Phys., 28, p. 513, 1909.

Before proceeding to the work that has been done on $\frac{e}{m}$ and v for negative carriers of electricity emitted from bodies either under the influence of ultra-violet light, or from bodies in an incandescent state, it might be well to say a few words about the velocity v. It must be noticed that v has always been determined by indirect methods, in a great many cases being deduced from a determination of $\frac{e}{m}$. That these indirect methods are however accurate has been proven by E. Wiechert[20] who in 1899 determined v by a direct and novel method.

In his method, cathode rays are passed through a diaphragm, S_1 the resulting thin pencil then traversing the discharge tube and passing through a second diaphragm S_2, then finally striking a screen at G. Between the cathode C and S_1 is a coil abcde wound around such that the cathode beam is deflected up or down. This coil is crossed by an



alternating current of high frequency, so that the cathode beam swings back and forth like a pendulum. By means of a permanent magnet M, the beam can be so deflected that the oscillating pencil of rays passes through the diaphragm only at the instant of maximum displacement when the velocity of displacement is zero.

Some distance from S_1 is a second diaphragm S_2 and a coil a'b'c'd'e' moveable along the tube, but connected to the first coil. For a certain position of this coil, the cathode particles enter its field $\frac{1}{4}$ or $\frac{3}{4}$ of a period after having crossed the first. The field

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20 Wied. Ann., 69, p. 739, 1899.

which was a maximum then is zero now, so that the second coil produces no deviation of the beam with the result that it passes through the second diaphragm undeviated, lighting up the screen G.

In moving the second coil along the tube one finds thus a series of neutral points, their distance apart representing the distance traversed by a cathode particle during a half period of the alternating field, the duration of which can be easily calculated from the equation

$$v = \frac{4L\lambda}{L}$$

where

V = velocity of light

4λ = distance between circuits for a whole period

L = wave length of waves surging through the two circuits

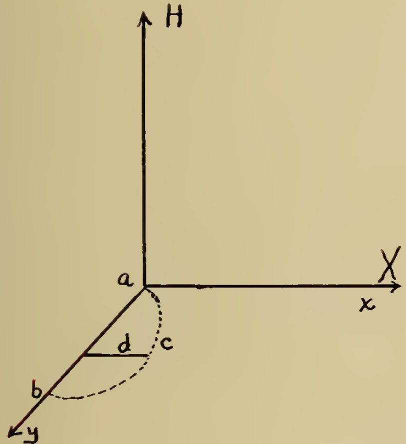
v thus determined gave an average value of 4.5×10^9

Knowing v, $\frac{e}{m}$ can be easily determined from the magnetic deflection, $\frac{e}{m} = \frac{v}{Hr}$. Wiechert found $\frac{e}{m} = 1.26 \times 10^7$ (mean value).

CHAPTER IV

$\frac{e}{m}$ and v for Negative Carriers Emitted from Objects Under Influence of Ultra-Violet Light

When ultra-violet light is incident upon a negatively charged metal plate, it has been found that negative carriers of electricity are emitted from the plate. By means of a properly directed magnetic field, the paths of these particles can be twisted into a curve, such that they return to the plate from whence they started.



Let H = direction of the magnetic field.

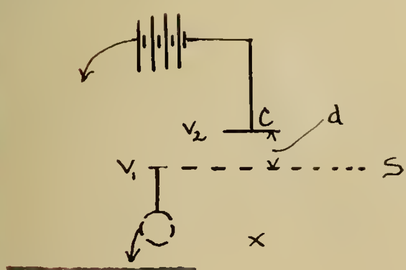
X = direction of the electric field.

Sir J. J. Thomson[21] has derived the equations of motion for these negative particles starting from the plane x = 0 at the time t = 0. If the electric and magnetic forces are uniform and at right angles to each other, then the position of a particle at any time t is given by the equations

$$x = \frac{m}{e}\frac{X}{H^2}\left\{1 - \cos\left(\frac{e}{m}Ht\right)\right\} \qquad (1)$$

$$y = \frac{m}{e}\frac{X}{H^2}\left\{\frac{e}{m}Ht - \sin\left(\frac{e}{m}Ht\right)\right\} .$$

The path of the particle will be a cycloid acb, in the xy plane, the



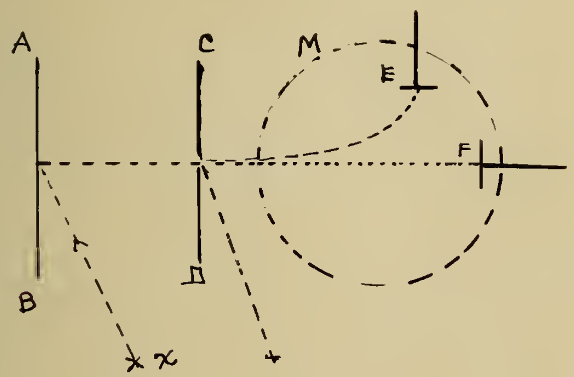
maximum distance of the particle from the x=0 plane being $d = \frac{2mX}{eH^2}$. In the apparatus used, C served as the negatively charged plate. Ultra violet light from x when incident upon

21 Cond. of Elec. through Gases, 2d. Ed., p. 113.

C (going through the grid S), caused the emission of these negative particles to the screen S which was connected to an electrometer.

It is evident from the above theory that all the particles coming from C will reach S if $d \leq \frac{2mX}{eH^2}$. For values of $d > \frac{2mX}{eH^2}$, there should be theoretically no charge delivered to S. Practically for a given value of X and H, if S is connected to an electrometer, and C gradually moved away, it will be found that for a certain value of d, the charge given to the electrometer commences to decrease. Were all the particles to start right from C and with the same velocity, the critical value for d would be sharp. Ordinarily this value for d is not a sharp one. Since $d = \frac{2mX}{eH^2}$, we can calculate $\frac{e}{m}$ from this equation.

Thomson's[22] method was not to vary d but to decrease the difference of potential between C and S, until he came to the point where upon application of the magnetic field, the charge given up to the electrometer would suffer a decrease. He found $\frac{e}{m} = 7.3 \times 10^6$.



P. Lenard[23] has also determined $\frac{e}{m}$ for particles emitted from a metal plate under the influence of ultra violet light.

Ultra violet light from a source x is incident upon the Al plate AB. The particles emitted from this negatively charged plate receive an acceleration from the accelerating condenser ABCD, and then are deflected by a magnetic field M, finally striking an electrode

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22 Phil. Mag., V., 48, p. 547, 1899.

23 Ann. d. Phys. 2, p. 359, 1900.

E connected to an electrometer. By varying the strength of the magnetic field, a maximum electrometer deflection is obtained, which shows that all of the negative particles are then falling upon E.

For the magnetic deflection we have the equation,

$$Hev = \frac{mv^2}{r} \tag{1}$$

from which r the radius of the path of the particles is known from dimensions of the apparatus.

For the kinetic energy imparted to the particles by the accelerating condenser, we have

$$\frac{1}{2}mv^2 = Ve, \tag{2}$$

where V is the potential difference between AB and CD. Lenard obtained for $\frac{e}{m}$ using the above equations,

$$\frac{e}{m} = 1.16 \times 10^7.$$

R. Reiger[24] employed Lenard's method for determining $\frac{e}{m}$ for the negative carriers emitted from glass under the influence of ultra violet light. The glass was charged negatively by means of a sheet of tin foil pasted on the back of the glass. He encountered difficulties in obtaining a maximum electrometer deflection for variations in H. This was due to variations in the intensity of the Hg discharge tube which he used as a source for ultra violet light. The value for $\frac{e}{m}$ which he obtained agreed however fairly well with Lenard's value. Reiger's value for $\frac{e}{m}$ was 1.07×10^7.

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24 Ann. der Phys., 17, p. 947, 1905.

CHAPTER V

$\frac{e}{m}$ and v for Negative Carriers Emitted from Hot Bodies

Long before the latter part of the 19th century it was known that a red hot metal renders the gas surrounding it conducting. Many qualitative observations were made, but it was not till the "eighties that any real work was done on this subject. In 1882, Elster and Geitel[25] found that Pt when heated to incandescence in a high vacuum emitted negatively charged particles. This has been more fully investigated theoretically as well as experimentally by O. W. Richardson[26].

Not only for Pt but for glowing bodies in general, it has been found that either positive or negative charges are imparted to surrounding bodies. The sign of electrification produced by glowing bodies, depends upon the nature of those bodies, the nature of the surrounding gas, the pressure of the gas, and upon the temperature of the glowing body. Hittorf[27] was one of the first to investigate the increased electrical conductivity imparted to gases by incandescent cathodes. Edison has also made a study of the subject in connection with his work on incandescent lamps.

It appears that at very high vacua, the particles or electrons seem to come right from the incandescent body. These negative carriers must be identical with those given off by an ordinary cathode, since $\frac{e}{m}$ has been found to be about the same in the two cases. If we regard all bodies and especially metals, as being full of a "gas" of negatively charged particles or electrons, these electrons being held within the body due to the attraction between the body and the electrons, then it appears that a high potential, ultra-violet light, or

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25 Wied. Ann., 16, p. 193, 1882.

26 Trans. Roy. Soc., 201 (A), p. 497, 1903.

27 Wied. Ann., 21, p. 119, 1884.

a high temperature, increases the kinetic energy of the electrons sufficiently, so as to permit the electrons to escape from the body.

In 1899 Thomson[28] determined $\frac{e}{m}$ for negative particles emitted from an incandescent carbon filament. His method was that employed for the case of ultr-violet light. Instead of the plate and grid below it, he employed two parallel Al disks, between which was placed a small semi-circular carbon filament parallel to the plates.

The theory and method employed was the same as before, the value of $\frac{e}{m}$ he found to be 7.8×10^6, a value agreeing very well with that obtained for cathode rays.

In 1904 G. Owen[29] determined $\frac{e}{m}$ for negative carriers from a Nernst filament, which is an oxide composed of the rare earths. He employed Thomson's method above, subjecting the discharged particles, to an electrostatic and a magnetic field placed perpendicularly to each other. The equation he employed was

$$\frac{e}{m} = \frac{2V}{d^2 H^2}$$

where V is the potential difference between filament and plate, the filament being charged to a high negative potential.

However, while Thomson varied V for a given magnetic field, Owen varied H for a given value of V, finding the smallest magnetic field for which a diminuition of the electrometer deflection would just occur.

He obtained for $\frac{e}{m}$ the value, 5.65×10^6.

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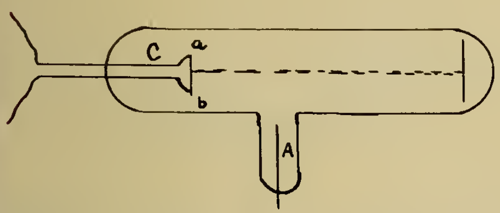
28 Phil. Mag., V, 48, p. 547, 1899.

29 Phil. Mag., VI, 8, p. 230, 1904.

CHAPTER VI

$\frac{e}{m}$ and v for Wehnelt Cathode Rays

Previously mentioned investigators have shown that negatively charged particles are emitted from incandescent Pt wires. A. Wehnelt[30] showed that the output of these negative particles could be enormously increased, by coating the Pt wire with a layer of any of the alkaline earth metal oxides, i.e., BaO, CaO and SrO. When used as a cathode, an enormous lowering of the potential drop is obtained at the cathode, so that a discharge can be kept up with only 20 volts potential difference between anode and hot lime cathode, for e.g., if CaO is used. The cathode dark space is supposed to represent a region poor in negative ions. If this impoverishment is prevented by the introduction in this space of a good source of negative particles, e.g., the hot lime or Wehnelt cathode as it is known, then the result is a lowering of the potential drop at the cathode.



Suppose a minute speck of CaO is placed on a piece of Pt foil ab which is held between two thick copper leads. On heating this Pt to incandescence and making this the cathode of a discharge tube, the fall of potential between the lime speck and anode A is much smaller than the fall of potential between the clean Pt and A. The result is that the whole cathodic discharge goes through the small lime speck, in the form of a thin sharp pencil of cathode rays of intensely blue color, the rays leaving the lime in a direction normal to the Pt surface. This type of a cathode is known as the Wehnelt cathode after its discoverer.

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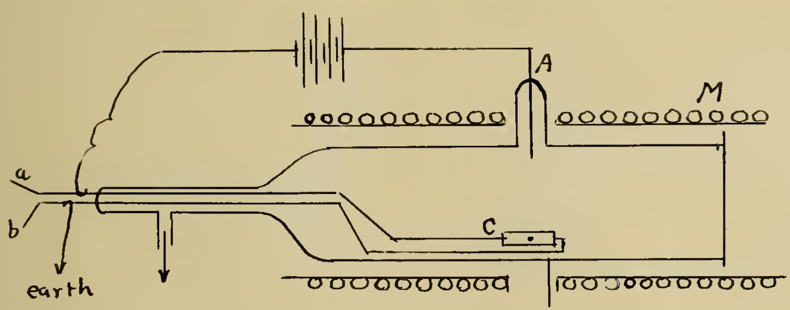
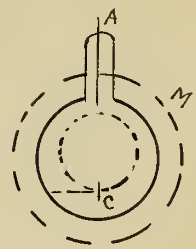
30 Verhand. d. Deut. Phys. Gesell., 5, p. 255-258, and 423 to 426, 1903; Ann. d. Phys., 14, p. 425, 1904; Phil. Mag., VI, 10, p. 80, 1905.

The Wehnelt cathode rays are very soft, the velocity of the rays being smaller than for ordinary cold cathode rays. The result is that these rays are easily absorbed by any residual air so that the vacuum must be very high to obtain a long beam. It thus becomes possible to send a cathode beam through exceedingly high vacua, vacua through which no cathode rays from a cold cathode could ever be made to pass. At a vacuum obtained by liquid air and charcoal, Dr. C. T. Knipp[31] has obtained Wehnelt cathode beams 60 cm. long.

These rays having such comparatively small velocities, are easily deflected by a magnetic field so as to lend themselves nicely to $\frac{e}{m}$ measurements. The magnetic deflection of the rays increases with increasing pressure within the discharge tube, keeping the temperature of the cathode constant. On the other hand, keeping the pressure constant, the deflection increases with rising temperature of the cathode.





Wehnelt[32] determined $\frac{e}{m}$ and v for these soft rays by means of magnetic deflections and from a knowledge of the potential drop between the electrodes, the latter being measured by means of a sounder placed between anode and cathode. The cathode beam issued from a small speck of CaO placed on a piece of Pt foil C which was heated electrically.

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31 Trans. A.I.E.E., p. 1883, 1912.

32 Ann. d. Phys., 14, p. 425, 1904.

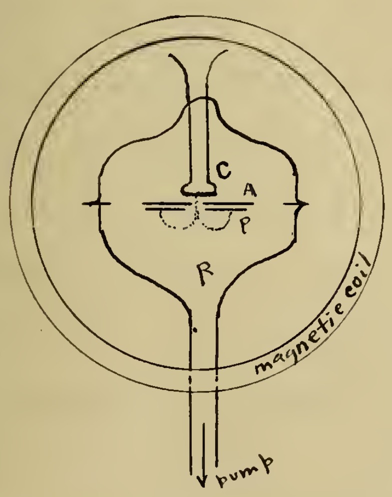
By means of a uniform magnetic field which surrounded the whole apparatus, the beam was twisted into the form of a circle, the diameter of which could be easily varied by varying the strength of the magnetic field. The plane of the circle was parallel to the plane of the flat end of the discharge tube, and the beam being so distinct, the diameter could be easily measured by sighting a telescope through the flat end. The equations used were those mentioned previously in connection with other workers, i.e.,

$$\frac{1}{2}mv^2 = Ve \qquad (1)$$

$$Hev = \frac{mv^2}{r} \qquad (2)$$

Wehnelt obtained an everage value of 1.48×10^7 for $\frac{e}{m}$, the values of v varying from 1.6×10^8 to 10.7×10^8 cm. per sec., variations in v being due to variations in the temperature of the Pt.

J. Classen[33] in 1908 used the photographic method for $\frac{e}{m}$, employing magnetic deflections and potential differences the same as Wehnelt.



A discharge chamber R was placed in a uniform magnetic field between two magnetic coils placed close together. The cathode beam issued from the Wehnelt cathode C and passing through the perforated anode A, was bent into a circle by means of the magnetic field, so that the beam described a half circle and struck a circular perforated photographic plate P mounted underneath the perforated circular anode A. The circle could be directed to the right or left depending upon the

33 Phys. Zeit., 9, No. 22, p. 762, 1908.

direction of the field. The diameter of the circle could be thus determined and by the use of the equations $\frac{1}{2}mv^2 = Ve$ and $Hev = \frac{mv^2}{r}$, $\frac{e}{m}$ could be determined.

The magnetic coils were mounted on a vertical axis, so that they could be rotated while the discharge chamber remained fixed. By thus rotating the coils, several exposures can be made by using the same photographic plate. The potential difference of discharge used was 1000 volts. The strength of the magnetic field was 56 Gausses, the diameter of the circle being about 37 cm.

Classen claimed that only 3 or 4 volts were required to cause a discharge at low temperatures of the Pt cathode, while only 1 volt was required at very high temperatures. Classen therefore concluded that very little energy of the electric field was used to cause the emission of the negative particles from the cathode.

Classen's value for $\frac{e}{m}$ was 1.773, $\times 10^7$ a value which he claimed correct to 2 in the 3rd decimal, a remarkable assumption in face of the Simon-Bucherer work. However as far as the results are concerned, Classen's value for $\frac{e}{m}$ compares most favorably with the best value for $\frac{e}{m}$ known, which is about 1.76×10^7.

So far only magnetic deflections and potential differences of discharge have been used in the determinations of $\frac{e}{m}$ for the Wehnelt cathode rays. However in 1912, Dr. C. T. Knipp[34] used electrostatic and magnetic deflections in his determinations, employing one of the methods used by earlier investigators for ordinary cathode rays. The cathode beam issued from C and struck a willemite screen S where it produced a phosphorescent spot. The magnetic and electrostatic fields

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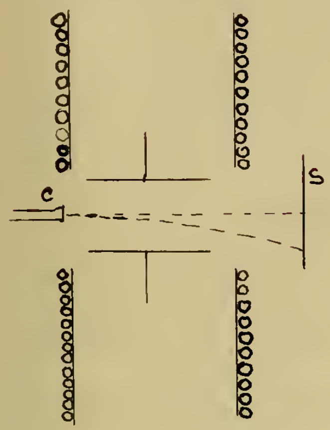
34 Trans. A.I.E.E., p. 1883, 1912.

were superimposed but parallel with each other, so that the magnetic and electrostatic deflections were at right angles to each other.



From the equation of motion of an electron through an electric field, i.e.,

$$m\frac{d^2y}{dt^2} = Ye,$$

he obtained for the electrostatic deflection

$$y = \frac{Ae}{mv^2} \qquad (1)$$

where A is a constant depending upon the strength of the field and the dimensions of the apparatus. The magnetic deflection is given by

$$z = \frac{Be}{mv} \qquad (2)$$

where B is another constant depending upon the dimensions of the apparatus and the magnitude of the magnetic field.

The magnetic field was not uniform for the whole path of the beam, and the resulting magnetic intensity was determined by Thomson's[35] triangle method.

Knipp's values for $\frac{e}{m}$ and v were respectively 1.5×10^7 and 1.6×10^9.

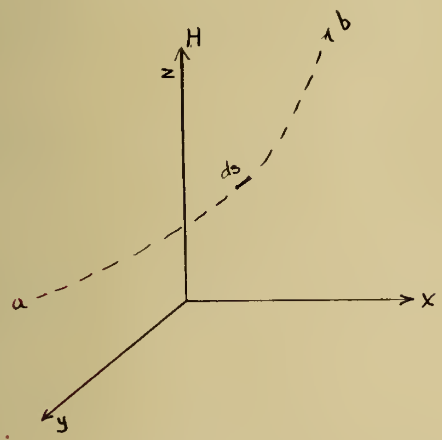
35 Phil. Mag., VI, 18, p. 844, 1909.

CHAPTER VII

Present Investigation

In the present investigation, the method employed was the utilization of the principle of both magnetic and electrostatic deflections. This avoids the assumption that all of the electric energy of the field is transferred into kinetic energy of the moving electrons. The fact that the beam of cathode rays issuing from the small speck of CaO, is so compact and well defined under proper conditions, makes it possible to introduce the cathode right in the center of an electric field without the usual encumbrances of diaphragms that are necessary when using a cold cathode. If we place the whole apparatus in a uniform magnetic field, then we are enabled to work in a uniform magnetic and in a uniform electrostatic field.

Theory



Consider an electron moving along the path ab under the influence of the magnetic field H whose direction is along the z axis.

Let

e = charge on the electron

m = mass of the electron

$\frac{ds}{dt}$ = velocity of the electron

The electric charge e with the above velocity is equivalent to a current $e\frac{ds}{dt}$. The direction cosines of the element of path of the electron, ds, is $\frac{dx}{ds}$, $\frac{dy}{ds}$ and $\frac{dz}{ds}$. Hence the components of the current $e\frac{ds}{dt}$ along the 3 axes, are, $e\frac{ds}{dt}\frac{dx}{ds}$, $e\frac{ds}{dt}\frac{dy}{ds}$ and $e\frac{ds}{dt}\frac{dz}{ds}$, or $e\frac{dx}{dt}$, $e\frac{dy}{dt}$ and $e\frac{dz}{dt}$.

From the fundamental law of electromagnetism, we have for the mechanical force acting on the electron in the X direction,

$$dF_x = m\frac{d^2x}{dt^2} = -He\frac{dy}{dt}$$

Similarly,

$$dF_y = m\frac{d^2y}{dt^2} = He\frac{dx}{dt}$$

and

$$dF_z = m\frac{d^2z}{dt^2} = 0 .$$

Since there is no mechanical force acting on the electron in the direction of H, i.e., along z.

Suppose that in connection with the magnetic field, we have an electrostatic field of constant strength E, acting along the direction parallel to the xz plane. We therefore have no component of E along the y axis. However along the x and z axes we have for the components of the electric force, X and Z. Hence the mechanical force acting on an electron in the x direction is

$$m\frac{d^2x}{dt^2} = Xe$$

Along y,

$$m\frac{d^2y}{dt^2} = 0$$

Along z,

$$m\frac{d^2z}{dt^2} = Ze$$

Combining these equations with those for the motion of the electrons under the magnetic field, we have the following equations of motion of the electrons under the influence of the magnetic and electrostatic fields,

$$m\frac{d^2x}{dt^2} = Xe - H\frac{dy}{dt} \tag{1}$$

$$m\frac{d^2y}{dt^2} = He\frac{dx}{dt} \tag{2}$$

$$m\frac{d^2z}{dt^2} = Ze \tag{3}$$

Integrating equations (1) and (2) we have,[36]

$$x = \left(\frac{X}{H} - v_o\right)\left(\frac{1 - \cos\omega t}{\omega}\right) + \frac{u_o}{\omega}\sin\omega t \tag{4}$$

$$y = \frac{u_o}{\omega}(1 - \cos\omega t) + \frac{X}{H}t + \left(v_o - \frac{X}{H}\right)\frac{\sin\omega t}{\omega} \tag{5}$$

where u_o and v_o are the initial velocities of projection of the electron along the x and y axes, and $\omega = \frac{He}{m}$.

If the direction of both magnetic and electrostatic fields are along z, then $X = 0$, and we have,

$$\left(x + \frac{v_o}{\omega}\right)^2 + \left(y - \frac{u_o}{\omega}\right)^2 = \frac{v_o^2 + u_o^2}{\omega^2}. \tag{6}$$

This equation is the equation of a circle on the XY plane and shows us that the projection of the path of the electron on the xy plane is a circle.

Integration of equation (3) gives us,

$$z = \frac{1}{2}\frac{Ze}{m}t^2 + \omega_o t \tag{7}$$

where ω_o is the velocity of projection of the electron along the z axis.

Since z increases with the square of the time, we thus see from equations (6) and (7), that the path of the electron would be a helix of increasing pitch, its axis coinciding with the z axis.

This helix has been experimentally realized as follows. Consider an electrostatic field produced between two large circular parallel Al plates, the direction of the field being vertical. Let a Wehnelt cathode be introduced between the electrostatic plates, the whole discharge chamber being surrounded by two large magnetic coils so that the lines of magnetic force are parallel to those of the electrostatic field. The cathode beam issues perpendicular to lines of force of both

36 J. J. Thomson, Cond. of Elec. Through Gases, 2d. Ed., p. 112.

fields, so that under the influence of a magnetic field only, the cathode beam will be bent into a circle of any desired size between the electrostatic plates. On application of the electric field, the circle will be drawn out in the form of a helix whose axis is also vertical. From the radius of the circle, pitch of the helix, and strength of the electric and magnetic fields, $\frac{e}{m}$ and v can be determined.

Let the Wehnelt cathode beam issue from the cathode in a direction perpendicular to the lines of magnetic and electric force. With no fields on, the beam travels in a straight line. When the magnet field is set up, the cathode beam is deflected into the form of a circle, the rays leaving the front side of the cathode and entering at the same point on the rear side of the cathode after its trip around the circular path.

The magnetic field of strength H exerts on an electron a force of attraction Hev towards the center of the circle. This is balanced by the centrifugal force of the electron $\frac{mv^2}{r}$, where

v = velocity of the electron in the direction of projection

r = radius of the circle

Therefore

$$Hev = \frac{mv^2}{r} \tag{1}$$

Therefore

$$v = H\frac{e}{m}r. \tag{2}$$

Let us consider the electrostatic field whose lines of force are in the z direction, and which has a constant field strength Z. This field exerts a force Ze on an electron, so that it is accelerated in the z direction with a force,

$$m\frac{d^2z}{dt^2} = Ze \tag{3}$$

Integrating this expression, z and $\frac{dz}{dt}$ being equal to zero when t = 0, we have,

$$z = Z\frac{e}{m}\,\frac{t^2}{2} \tag{4}$$

The time t taken by the electron to move once around the circle with constant linear velocity in the horizontal plane is equal to the time taken by the electron to move once around along the helical path. This same condition of things is met with in the horizontal projection of projectiles, where the time taken for the projectile to strike the ground is the same as the time taken by the projectile to move along the horizontal path with the constant initial horizontal velocity.

Therefore

$$v = \frac{2\pi r}{t}$$

for one revolution of the electron. If the electron moves n times around on the helical path before striking the upper electrostatic plate, then,

$$v = \frac{2\pi rn}{t} \tag{5}$$

or

$$t = \frac{2\pi rn}{v}$$

Substituting this value of t in equation (4)

$$z = \frac{Ze}{m}\,\frac{4\pi^2 r^2 n^2}{2v^2} \tag{6}$$

Therefore

$$v^2 = Z\frac{e}{m}\,\frac{2\pi^2 r^2 n^2}{z} = H^2\,\frac{e^2}{m^2}\,r^2$$

from equation (2)

Therefore

$$\frac{e}{m} = \frac{Z}{H^2}\,\frac{2\pi^2 n^2}{z} \tag{7}$$

The value of n employed in this work, as will be later explained was $\frac{1}{2}$. Hence the equation employed in the determination of $\frac{e}{m}$ was,

$$\frac{e}{m} = \frac{Z}{H^2}\frac{\pi^2}{2z} = \frac{V}{dH^2}\frac{\pi^2}{2z} \tag{8}$$

where d = distance between the electrostatic plates and V = their potential difference.

To determine the velocity v of the electrons, we have from equation (1),

$$\frac{e}{m} = \frac{v}{Hr}$$

Also from equation (6)

$$\frac{e}{m} = \frac{v^2 z}{Z2\pi^2 r^2 n^2}$$

Equating these two expressions for $\frac{e}{m}$, and solving for v,

$$v = \frac{Z2\pi^2 rn^2}{H\ \ z}$$

For

$$n = \frac{1}{2}$$

$$v = \frac{Z\pi^2 r}{2Hz} = \frac{V\pi^2 r}{2dHz} \tag{10}$$

v is thus determined independent of $\frac{e}{m}$. We could of course determine v from a knowledge of the discharge potential and of the value $\frac{e}{m}$, since,

$$\frac{1}{2}mv^2 = Ve \tag{11}$$

Therefore

$$v = \sqrt{\frac{V}{2}\frac{e}{m}} \tag{12}$$

where V is the discharge potential, i.e., potential between anode and cathode.

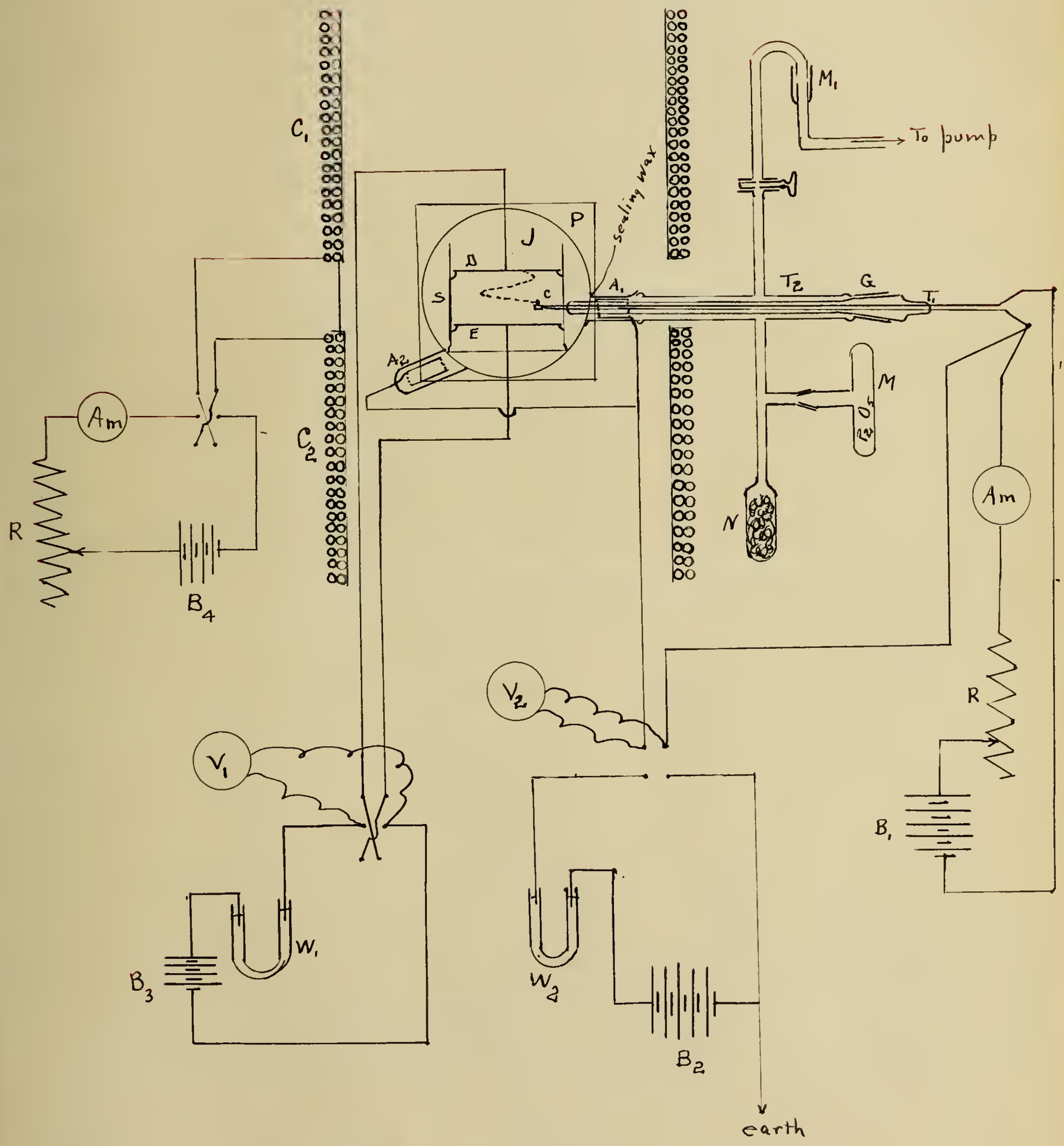
Description of the Apparatus

The Discharge Chamber

The discharge chamber consisted of a jar J, 21 cm. wide and 18cm. deep, lying on its side. The mouth of the jar was ground plane by means of emery and then polished smooth with rouge, to receive a square piece of plate glass P, which was fitted in air tight by means of an equal part mixture of beeswax and rosin. The plate glass permitted

DIAGRAM OF APPARATUS

(Front View)



measurements of the helix to be taken accurately without any optical distortion.

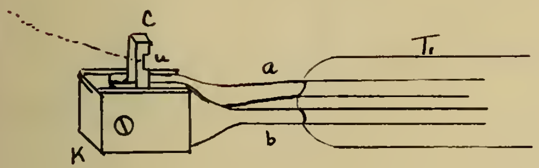
The Electrostatic Field

The electrostatic deflections were affected by two parallel Al plates, D and E, resting on am improvised glass support S. The plates were circular being 15 cm. in diameter. The lead wires from the plates were conducted through small holes bored out through the walls of the jar, to a source of high potential, B_3, which consisted of a great number of small accumulators all in series. The upper electrostatic plate D was connected to the (+) pole of the battery through a water rheostat W_1, the lower plate being connected to the (-) pole of the battery. This arrangement allowed of the drawing out of the circle into a helix, in an upward direction. The difference of potential between the two plates was measured by means of a Kelvin multicellualar voltmeter, placed across the lead wires of the electrostatic plates.

Throughout the whole work the electrostatic plates were placed reasonably close together for the obtaining of a uniform field, the distance between the plates varying from 4 to 8 cm. The plates could not be placed much closer on account of the conditions of the problem. In the first place the electrostatic deflections were fairly large, i.e., from $\frac{1}{2}$ to 2 cm., for the first half turn. Ample room was therefore required to permit the existence of the helix between the plates for at least a little over one half turn. Had the plates been closer together, and very weak fields employed, then the resulting deflections would not have been large enough to admit of accuracy, since it must be remembered that the beam is not a geometrical line, but has some thickness.

The Wehnelt Cathode

The Wehnelt cathode C, was introduced between the plates through a hole 2.5 cm. wide in the side of the jar bored out for this purpose.



The 2 lead wires (a) and (b) which served to heat up the Pt strip C to incandescence, were conducted through a glass tube T_1 which fitted into an outer sleeve T_2 by means of a ground joint G. The ends of the lead wires were flattened out and by means of the screw clamp K, the Pt strip C was mounted. The employment of properly placed thin mica sheets directed the current through this bent Pt strip. To localize the incandescence an indentation, u, was made at the place where the small lime spot was located. This method of mounting the Wehnelt cathode is due to Dr. C. T. Knipp[37] and served excellently in this work since the mounting was rigid and allowed of a quick change in the Pt strip. The ground glass joint G permitted the rotation of the cathode so that the cathode beam could be oriented around till its path was perpendicular to the lines of magnetic force. This joint therefore admitted of easy adjustment in obtaining a circular path.

The two lead wires (a) and (b) were connected through a resistance R and ammeter Am to a storage battery B_1 which served to supply the heating current.

The discharge potential was furnished by a high potential battery B_2 furnishing 1000 volts. The cathode was connected to the (-) pole of this battery and was also grounded. The (+) pole of this battery was connected to the anode A_1 through a water resistance W_2. This

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37 Phys. Rev., p. 58, Jan. 1912.

anode was simply an Al collar on the inside of the tube T_2. A_2 served as a sort of an auxiliary anode which could be connected or disconnected at will. To measure the discharge potential, a Braun electrostatic voltmeter was used.

Connected with the tube T_2 was a drying tube M containing P_2O_5 which served to keep the discharge chamber in a continual dry state. The charcoal tube N served to bring the vacuum up to a remarkably high degree when immersed in a Dewar flask containing liquid air.

The discharge chamber was evacuated by means of a Gaede pump admitting of quite a high vacuum. This pump was connected to the apparatus through a Hg seal M_1.

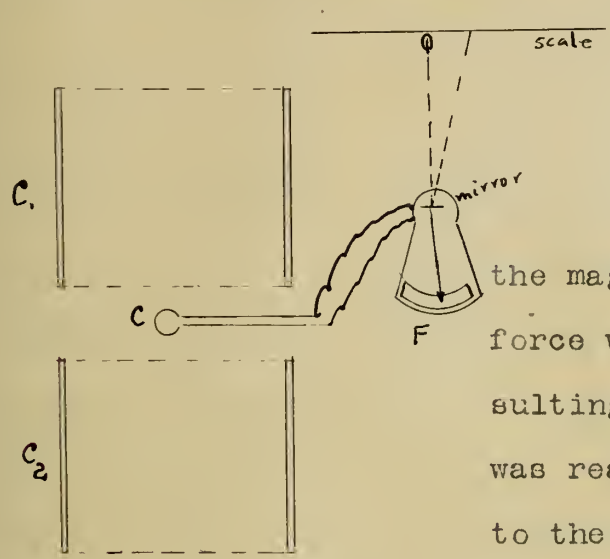
The small lime spot on the Pt strip of the cathode was affected by the placing of a small particle of Bank of England sealing wax on the Pt then electrically heating the Pt gradually up to redness. The sealing wax was thereby changed to a small round speck of CaO, whose diameter varied from $\frac{1}{2}$ to 1 mm.

The Magnetic Field

A uniform magnetic field was furnished by two large vertical coils, C_1 and C_2, which enclosed the whole apparatus. The coils were about 41 cm. in diameter and each 30 cm. high. By means of proper supports, they were separated from each other by a vertical distance of 8.5 cm. This admitted the introduction of the cathode into the discharge chamber, and allowed observations to be taken on the cathode rays within the chamber. The coils consisted of two layers of fairly thick wire allowing a little over 6 turns per cm. Battery B_4 served to supply the current which was accurately read by a Siemens-Halske ammeter.

To obtain the value of the magnetic intensity for any given value

of the current, a Grassot fluxmeter was employed. This is an instrument which gives the total flux or number of lines through a standard auxiliary coil 5.57 cm. in area and containing 100 turns of wire. To find the flux through the auxiliary coil the glass apparatus within the magnetic coils was removed and the coil C was properly supported



in the region occupied by the circular cathode beam. The coil C was laid horizontally so that on breaking or making the magnetic circuit, the lines of force would thread the coil C. The resulting deflection of the fluxmeter, F was read by means of a mirror, attached to the moving coil of the flux meter, and by a lamp and scale arrangement, the flux meter carrying a mirror for more accurate readings.

By sending currents of various strengths through the magnetic coils and noting fluxmeter deflections for each current, it was possible to plot a curve showing the relation between the current and the corresponding deflectionsof the fluxmeter as read with the aid of lamp and scale.

The value of a scale division in terms of the scale divisions on the flux meter itself was found by reading the lamp and scale deflection, and also the deflection of the needle of the fluxmeter for a given current. Table I gives the value of 1 cm. on the lamp and scale arrangement in terms of divisions on the flux meter itself, each division being equal to 10000 Maxwells, one Maxwell being equal to the to-

total flux through the auxiliary or test coil C.

TABLE I

Current in amp.	Scale deflection	Divisions on fluxm.	Value of 1 cm. in terms of Flux.	Mean
13.0	6.67	4.08	0.6117	
12.5	6.11	3.93	0.5897	
3.8	1.95	1.15	0.6708	
5.98	2.94	1.97	0.6136	
7.92	3.90	2.39	0.6432	
				0.6258

Table II shows the variation of H in terms of lamp and scale deflections, with the current i in amperes. The deflections are in cm. Four values were obtained for each value of the current i.

From Tables I and II, and from the straight line curve between current and scale deflection of fluxmeter, we can derive the following empirical equation between H and i. Since H = 0 when i = 0, and since the curve is a straight line through the origin, we have

H = Ki where K is the tangent of the curve

= 0.5082 in terms of scale deflections

or 0.5082 x 0.6258 in terms of divisions on the fluxmeter. But each division is 10000 Maxwells which is equal to $\frac{10000}{5.57 \times 100}$ Gausses when translated into magnetic intensity H. It will be remembered that 5.57 is the area of the coil and 100 = number of turns. Hence $\frac{10000}{5.57 \times 100}$ = number of lines per sq. cm. = H in gausses

Therefore

$$H = \frac{0.5082 \times 0.6258 \times 10000}{5.57 \times 100} \times i\ .$$

This gives H in absolute units, i being in amperes.

That the magnetic field over the region between the electrostatic

plates was constant, is apparent not only from the dimensions of the magnetic coils, but also from the fact that the path of the cathode beam in the magnetic circle was a perfect circle as far as the eye could judge. However to verify this qualitative observation , a quantitative observation of the constancy of the magnetic field was taken. The method employed was simply to place the small test coil at various positions from the center of the magnetic coils and note the fluxmeter deflections for various distances of the coil from the axis of the magnetic coils. The lamp and scale arrangement was again used to note more accurately a change in H if there were any change.

It was found that for a region of about 16 cm. diameter, the magnetic field was practically constant. Since the maximum diameter of the circular path of the cathode rays employed was only about 12cm. hence we see that we have for this region a perfectly constant field. Table III shows the variation of H in terms of scale deflections with variation of the distance of the test coil from the axis of the magnetic coils. The strength of the current used was 6 amperes. The first column of the table represents the distance of the test coil from the axis of the coils. The results of this table are plotted on the following curve, showing the constancy of the deflection with varying distance from the center.

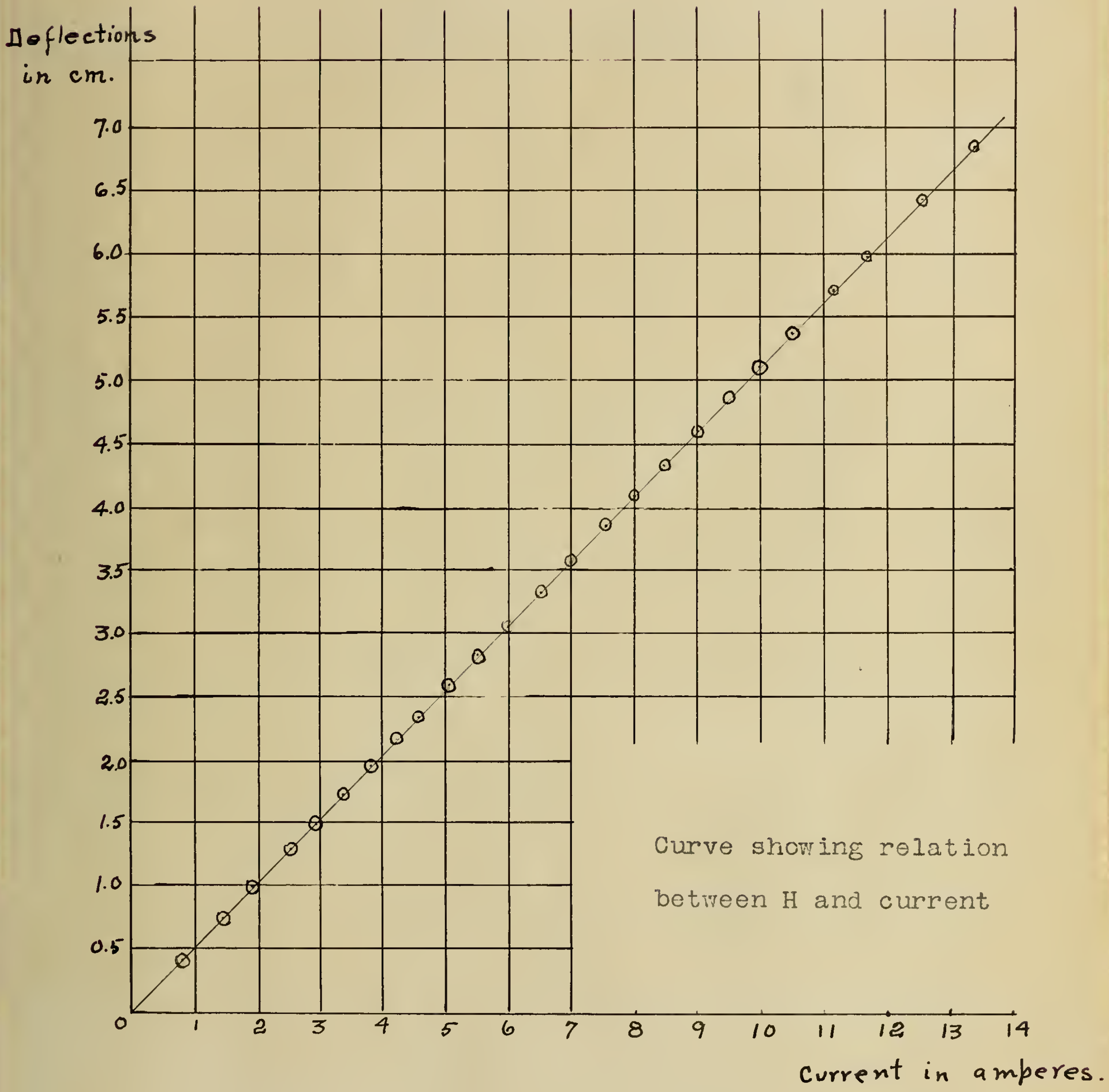
TABLE II

i	Deflections			i	Deflections		
	Breaking Circuit	Closing Circuit	Mean		Breaking Circuit	Closing Circuit	Mean
0.82	0.44 0.40	0.39 0.40	0.41	7.00	3.54 3.53	3.56 3.59	3.56
1.46	0.78 0.75	0.73 0.74	0.75	7.51	3.80 3.76	3.84 3.86	3.82
1.91	1.00 0.98	0.98 0.98	0.99	8.00	4.06 4.01	4.10 4.07	4.06
2.49	1.23 1.22	1.30 1.26	1.25	8.47	4.31 4.29	4.30 4.30	4.30
2.90	1.47 1.49	1.48 1.48	1.48	9.00	4.53 4.55	4.61 4.62	4.58
3.37	1.74 1.73	1.70 1.71	1.72	9.48	4.77 4.80	4.85 4.83	4.82
3.78	1.91 1.90	1.95 1.95	1.93	10.00	5.06 5.10	5.10 5.11	5.09
4.20	2.15 2.17	2.12 2.13	2.14	10.50	5.31 5.35	5.40 5.41	5.37
4.54	2.30 2.31	2.32 2.30	2.31	11.11	5.60 5.61	5.70 5.72	5.68
5.03	2.55 2.55	2.59 2.56	2.56	11.70	5.90 5.92	6.00 5.99	5.95
5.50	2.79 2.81	2.81 2.82	2.81	12.57	6.39 6.40	6.45 6.46	6.43

(Continued on next page)

TABLE II continued

i	Deflections			i	Deflections		
	Breaking Circuit	Closing Circuit	Mean		Breaking Circuit	Closing Circuit	Mean
5.99	3.03 3.02	3.07 3.02	3.04	13.33	6.77 6.72	6.87 6.82	6.80
6.49	3.30 3.27	3.30 3.32	3.30				

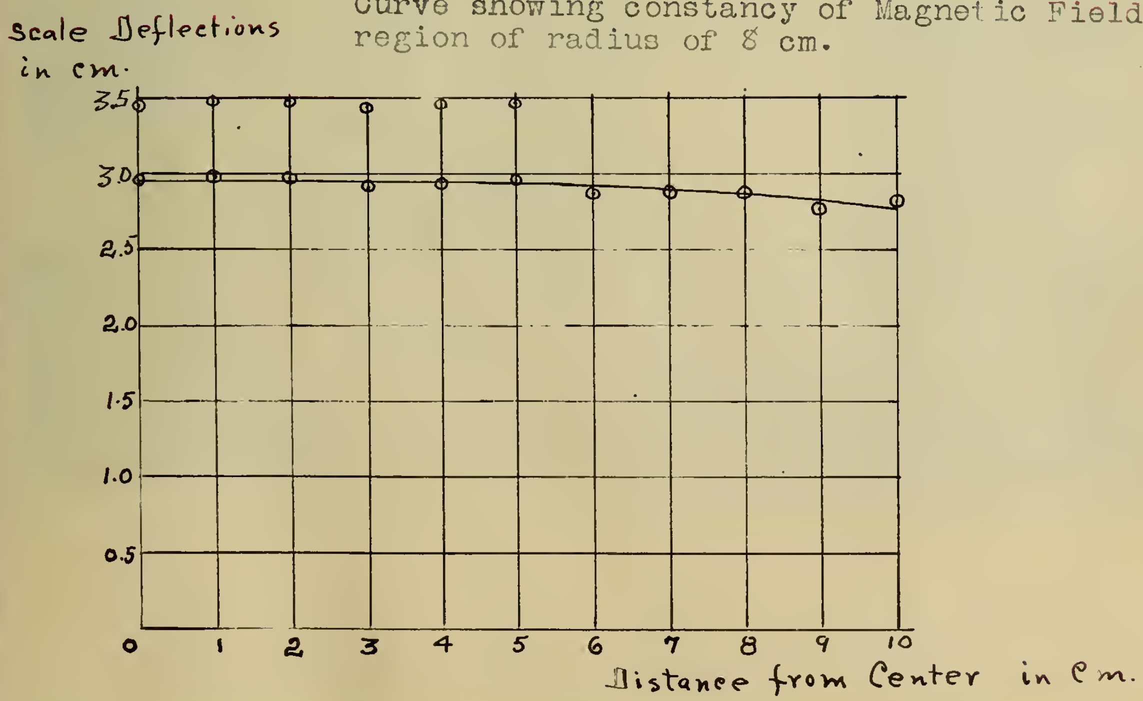


Curve showing relation between H and current

TABLE III

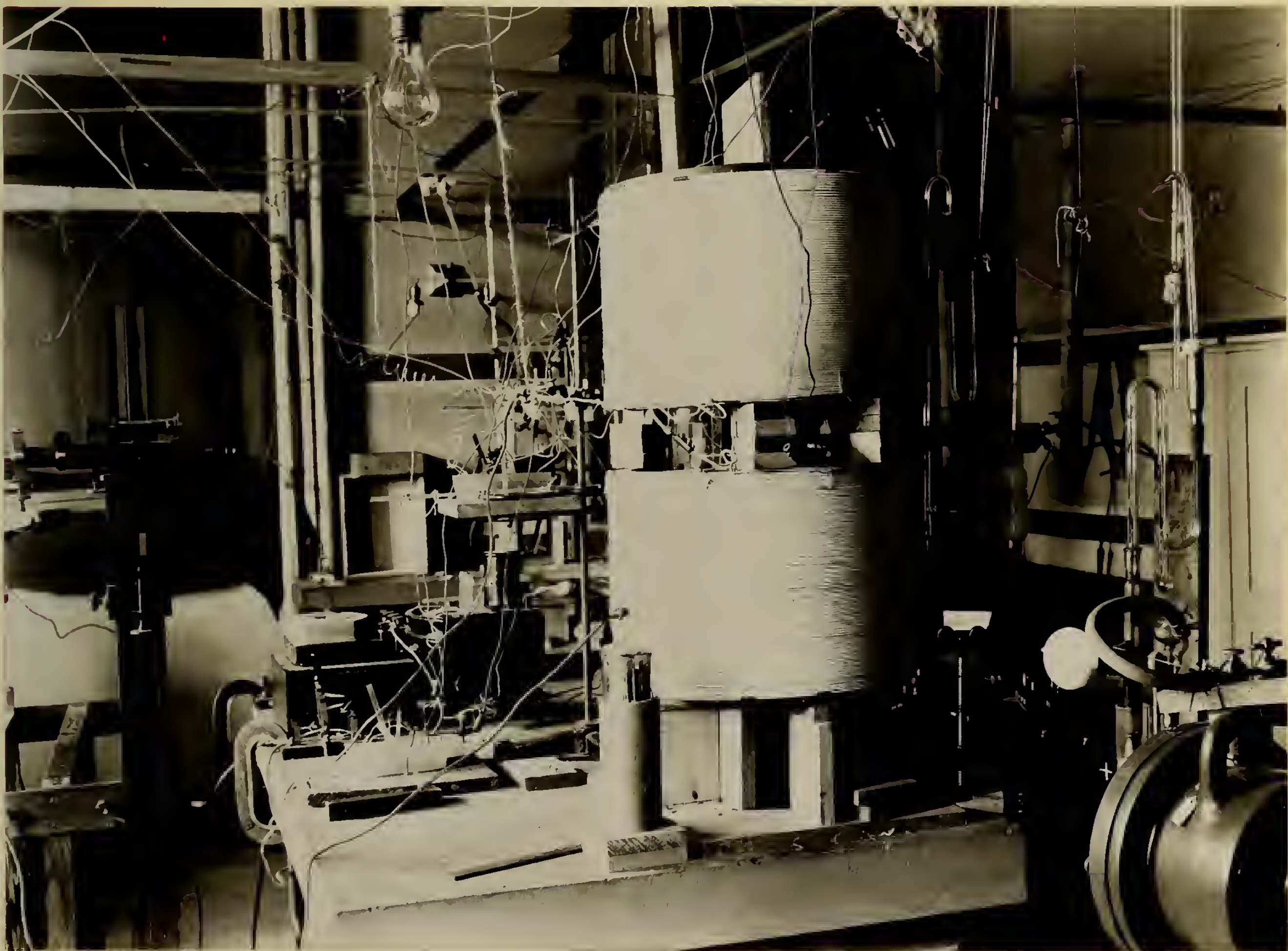
Distance	Making Circuit	Breaking Circuit	Mean
0 cm	3.02	2.90	2.96
1	3.03	2.94	2.99
2	2.98	2.98	2.98
3	3.02	2.83	2.93
4	3.00	2.89	2.95
5	3.00	2.93	2.97
6	2.97	2.81	2.89
7	2.94	2.83	2.89
8	2.91	2.86	2.89
9	2.84	2.71	2.78
10	2.86	2.75	2.81

Curve showing constancy of Magnetic Field over region of radius of 8 cm.



PHOTOGRAPH OF APPARATUS

(Cathode through rear of Discharge Chamber)



Preliminary Part of the Work

The preliminary part of this work consisted in testing out the workability of this type of apparatus for the measurement of $\frac{e}{m}$ by the helical method. The original apparatus was much smaller than the one described. The jar which acted as the discharge chamber was placed in a vertical position, and the distance between the electrostatic plates could be adjusted from an external point by means of a ground glass joint.

The original intention of the work was to have the cathode located between the electrostatic plates, and then by varying the distance between these plates by means of a revolving glass windlass through a ground glass joint, to obtain a whole number of turns between the cathode and upper plate. From the distance between cathode and upper plate and from the number of turns of the helix over the same distance, it was thought possible to determine $\frac{e}{m}$. It was however soon found that this method was not applicable for the reason that the cathode beam after having traversed the helical path the first time around, would receive a great upward distortion on passing over the cathode on its journey around the second turn of the helix. This of course is apparent from the fact that both the electrons composing the beam and the cathode itself, are negatively charged. The distortion undergone by the rays was therefore such that the regularity in increase of the pitch of the helix was disturbed and hence $\frac{e}{m}$ could not be determined from a large number of turns.

It was therefore thought advisable to measure the upward deflection of only the first one-half turn, before the rays received any distortion. To this end, the apparatus was made larger and more

perfect than the first. A plate glass window was also employed to avoid optical distortion of the beam to the sides of the jar. The larger electrostatic plates employed and the larger and deeper magnetic coils insured more uniform fields.

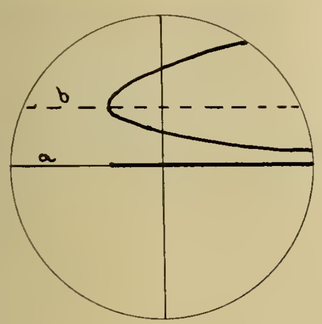
Method of Procedure

The actual mode of procedure in making measurements was as follows. Since the Wehnelt cathode rays are obtainable with distinctness only at a high vacuum, the exhaustion was carried on to a very high degree by means of the Gaede pump. This exhaustion was continued for over one hour till the glass supports within the discharge chamber phosphoresced brightly under the influence of an induction coil discharge through the vacuum. Liquid air was then applied to the charcoal tube and the vacuum thus raised still higher. In fact the vacuum would reach such a high point, that an ordinary induction coil discharge could not be made to pass through the discharge chamber, the discharge leaping across the terminals of the commutator outside of the discharge chamber. It is thus apparent that the effect of residual air in the chamber upon the cathode beam was negligible.

The Wehnelt cathode beam was started by closing the constant potential discharge circuit of 1000 volts, and then heating the Pt strip up to dull redness. It usually took some time for the beam to appear. Closing and breaking the magnetic circuit sometimes aided in hastening the appearance of the beam. Usually it was thought advisable to get the beam started before application of liquid air, for at a higher vacuum the beam was less prone to appear than at lower vacua. However once the beam appeared, the discharge could be stopped, liquid air applied and the apparatus allowed to stand thus for 15 minutes or so, the beam then appearing very quickly at a higher vacuum when the discharge circuit was closed.

The first appearance of the beam was in the form of a faint whitish cloud. Since measurements on the beam with the cathetometer required distinctness, hence the Pt strip was heated hotter to obtain a sharper beam. When first starting a series of measurements, the Pt was not heated to a very high temperature since the lime cathode seemed to lose its efficiency to send out a sharp beam, with increased heat and continued use. It was thought that the less intense the heat the longer the lime on the hot Pt would last, before it would gradually disappear or sputter off from the hot Pt.

Having raised the temperature of the Pt sufficient to produce a fairly sharp beam, the magnetic field was turned on twisting the beam into a circle, or as usually was the case, the beam would strike the lines of force at an angle less than 90°, with the result that the beam was not in the form of a plane circle but in the form of a helix. The cathode was then rotated in the ground glass joint until the helix

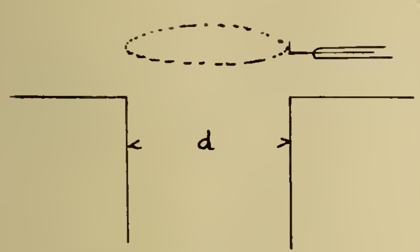


was degraded into a perfect circle. Looking through the cathetometer, the cross wire (a) was set in the plane of the circle. On turning on the electrostatic field, the circle was changed into a helix and the horizontal cross wire was then placed at (b) to measure the deflection for a half turn. The difference in the two readings of the cathetometer gave the electrostatic deflection z.

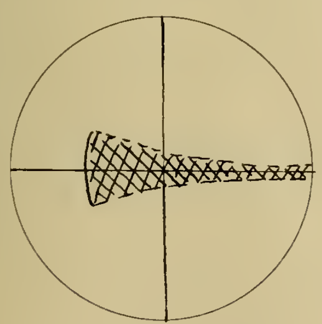
The difference of potential between the electrostatic plates was then read off from the voltmeter. The current in the magnetic coils was read off from an ammeter.

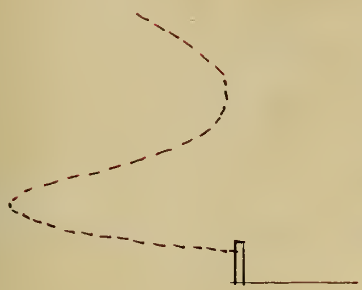
It will be remembered that the formula for the velocity of the rays required the radius of the circular path of the beam. This

necessitated the measurement of the diameter of the circle which was done as follows. Viewing the circle through the plate glass window along the plane of the circle, the latter appeared as a thick line going out a certain distance from the cathode. To obtain the length of this line, i.e., the diameter of the circle, a square was employed. One leg of this square was placed against the plate glass window, and by sighting along the other leg of the square, the latter was moved along till its edge coincided with one edge of the circle when viewed horizontally. The same was done for the other side of the circle, a point being marked on the glass each time. The distance d between these points is the diameter of the circle. By working carefully measurements taken were good to within 0.5 mm. per 6 or 7 cm. This was as close as could be expected for the beam is not a geometrical line.



Sharp beams were not always obtainable, especially in the first part of the work. With the beam oriented around till it described a perfect circle in the magnetic field, the beam would sometimes have this appearance at the point of a half turn from the cathode, when viewed through the cathetometer along the plane of the circle. It is thus apparent that it was no easy matter to know just where to set the horizontal cross hair of the cathetometer. With the electrostatic field on, the beam was always thrown into a more compact form having this appearance as it issues from the cathode C and goes into the helical form.





Since the accuracy of the results depends a good deal on the sharpness of the beam, attempts were made to see under what conditions it could be made the sharpest. Various methods of putting on the lime spot were tried. Even a concentrated solution of CaO was tried, instead of using the sealing wax. A small drop of this solution was placed on the Pt and the latter then heated slowly to redness. But this proved unsatisfactory.

Lower discharge potentials than 1000 volts were employed, but it was found that although a discharge could be obtained at much lower than 1000 volts, still the beam was not so distinct and distinctness was what was desired.

After many trials, it was finally found that the best beams obtainable were those from a very small spot of lime using a very small particle of sealing wax. Also it was found that heating up the Pt immediately to a fairly high temperature gave a much better beam than by using a much lower temperature, even though the lime spot would not hold out so long.

With this condition of affairs very compact and bright beams were obtainable, and results were obtained which are more consistent and reliable than those first obtained. These results are in Table V while the first values of $\frac{e}{m}$ obtained are found in Table IV.

In all cases there was a slight contraction in the diameter of the circle when drawn out into the form of a helix.

Modifications

There were two other modifications which were introduced in this work, which though not seeming to have given very consistent results

are however worth mentioning. It was at first thought that the introduction of the cathode between the plates must result in some distortion of the electrostatic field. It was therefore deemed desirable to place the cathode beneath the lower plate and by means of a large enough slit to introduce the Pt part of the cathode above the lower

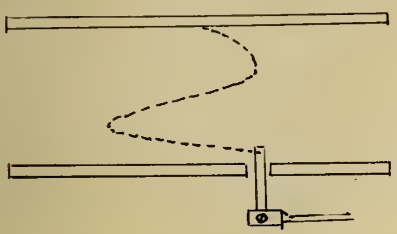


plate. The slit was about 2.5 mm. wide and 2 cm. long, and small enough compared to the dimensions of the plates so that very little distortion was introduced. The Pt part of the cathode was made long enough so that the cathode could be introduced tilted downwards through the glass sleeve T_2 (page 40a), and then by rotating the cathode in the ground glass joint, the Pt strip was swung into place with its top end containing the lime spot above the lower electrostatic plate. As before the cathode was oriented around till a perfect circle was obtained, the electrostatic field applied, and the deflection for the first half turn measured by means of the cathetometer.

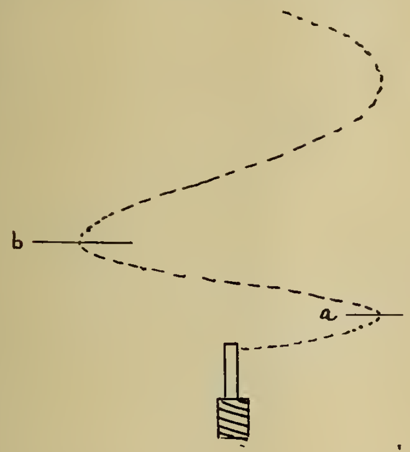
This modification did not however improve the general run of values obtained. It might be stated here, that although the cathode was grounded in the whole work, the lower or negative electrostatic plate could not however be grounded, for on grounding it an enormous electrostatic deflection of the cathode beam was obtained even though

the electrostatic field was not on. The beam as it left the cathode went straight up as here shown. The grounding of the lower plate placed it in connection with the cathode with the result that the whole lower plate was charged with a large negative charge. This induced a large positive charge on the upper plate with the result that a large electrostatic field was

established causing this enormous deflection.

Since the lower plate could not be grounded, hence a difference of potential must have existed between the cathode and the lower plate, thus causing some distortion of the beam as it left the cathode. The values obtained are contained in Table VI.

The other modification that was introduced in the apparatus was that of introducing the cathode through the back of the discharge chamber instead of through the side. This modification is shown clearly on the photograph. The cathode was introduced between the electrostatic plates. The reason for introducing this modification was this. It was just mentioned that there was some suspicion entertained that the beam left the cathode in a somewhat distorted state.



It was therefore thought that we might more nearly obtain the true deflection of half a turn by measuring on the helix the distance between a quarter turn and $\frac{3}{4}$ of a turn. This would give us the electrostatic deflection free of the cathode. The horizontal cross hair of the cathetometer was placed first at (a) and then at (b), the difference giving us the deflection for half a turn. The results are given in Tables VII and VIII.

It might be mentioned that in the above arrangement, a plate laid underneath the lower electrostatic plate, served as the anode.

Results

TABLE IV

Distance between the electrostatic plates = 7.78 cm.

	i	H	V	z	2r	$\frac{e}{m} \times 10^7$	$v \times 10^9$	
1	3.48	1.76	110	1.77	9.9	1.01	0.99	From red heat to white heat
2	3.47	1.75	82	1.36	8.6	0.99	0.84	
3	2.91	1.48	110	2.01	8.8	1.26	0.92	
4	2.80	1.42	37	1.10	7.7	0.84	0.51	
5	2.80	1.42	37	1.01	7.7	0.91	0.56	
6	3.40	1.73	78	1.20	7.8	1.09	0.83	
7	4.20	2.13	122	1.31	6.3	1.03	0.78	
8	3.30	1.67	75	1.11	8.5	1.22	0.97	
9	3.05	1.54	74	1.16	9.0	1.35	1.05	
10	2.90	1.47	78	0.85	10.3	2.13	1.81	
11	3.12	1.58	78	1.37	9.3	1.15	0.95	
12	3.50	1.77	162	1.58	7.6	1.64	1.24	
13	3.00	1.52	78	1.27	8.9	1.34	1.02	
14	4.11	2.09	205	1.57	5.8	1.50	1.03	
15	2.74	1.38	39.5	0.53	10.8	1.97	1.65	
					Mean	1.30	1.01	

v obtained from $v = \sqrt{2\frac{e}{m}V} = 1.61 \times 10^9$

where

$$\frac{e}{m} = 1.30 \times 10^7$$

$$V = 1000 \text{ volts}$$

Note: H found in these tables is in terms of scale divisions when using the mirror attachment on fluxmeter.

TABLE V

Distance between the electrostatic plates = 6.60 cm.

	i	H	V	z	2r	$\frac{e}{m} \times 10^7$	$v \times 10^9$	
1	3.30	1.67	81.3	1.01	10.1	1.71	1.62	From red heat to white heat.
2	3.99	2.02	81.2	0.85	7.7	1.39	1.21	
3	3.33	1.69	79.5	1.08	10.0	1.56	1.14	
4	4,70	2.39	119.5	0.98	6.4	1.27	1.09	
5	3.20	1.62	80.0	1.10	10.0	1.64	1.49	
6	3.75	1.91	119.3	1.33	7.7	1.46	1.20	
7	3.43	1.73	79.5	0.99	9.3	1.59	1.44	
8	2.90	1.47	120.0	1.94	10.2	1.70	1.43	
9	3.62	1.84	83.0	1.01	8.1	1.44	1.20	
10	2.90	1.47	120.0	1.72	10.3	1.91	1.63	
11	3.61	1.83	126.0	1.30	8.7	1.71	1.53	
12	3.35	1.69	43.0	0.43	8.9	2.07	1.75	
13	4.20	2.13	120.0	1.05	7.2	1.49	1.29	
					Mean	1.61	1.39	

v from $v = \sqrt{\frac{2e}{m} V} = 1.79 \times 10^9 \frac{cm}{sec}$.

TABLE VI

Distance between electrostatic plates = 4.37 cm.

	i	H	V	z	2r	$\frac{e}{m} \times 10^7$	$v \times 10^9$	
1	3.90	1.97	77.3	1.21	8.4	1.47	1.37	
2	3.80	1.93	40.8	0.65	9.0	1.51	1.47	From
3	3.70	1.86	75.0	1.76	9.0	1.10	1.04	red
4	4.80	2.43	83.2	1.03	6.2	1.22	1.04	
5	3.26	1.67	77.0	1.49	9.3	1.66	1.45	heat
6	3.34	1.71	64.0	0.89	7.9	2.20	1.67	to
7	3.32	1.69	65.5	0.84	8.3	2.44	1.92	
8	3.89	1.96	65.6	0.88	7.7	1.74	1.47	white
9	3.70	1.86	83.0	1.07	7.4	2.01	1.55	heat
					Mean	1.71	1.44	

Mean v from $v = \sqrt{\frac{2e}{m} V} = 1.85 \times 10^9 \frac{cm}{sec}$.

TABLE VII

Distance between electrostatic plates = 4.27 cm.

	i	H	V	z	2r	$\frac{e}{m} \times 10^7$	$v \times 10^9$	
1	4.40	2.24	62.5	0.88	6.0	1.30	0.98	From
2	4.20	2.12	84.0	0.76	8.5	2.25	2.28	red
3	3.62	1.83	66.0	1.76	10.4	1.03	1.10	heat
4	3.58	1.82	61.3	0.95	9.5	1.78	1.73	to
5	3.02	1.52	62.0	0.69	9.2	3.56	2.80	white
6	3.01	1.52	63.0	0.95	9.1	2.63	2.04	heat
7	3.90	1.97	64.0	1.06	7.8	1.42	1.23	
8	3.01	1.52	63.0	1.67	9.3	1.50	1.19	
					Mean	1.93	1.67	

Using 1000 volts as discharge potential.

$v = \sqrt{\frac{2e}{m} V} = 1.97 \times 10^9 \frac{cm}{sec}$.

Distance between the electrostatic plates = 6.38cm.

	i	H	V	z	2r	$\frac{e}{m} \times 10^7$	$v \times 10^9$	
1	5.39	2.74	87.3	0.83	7.1	0.86	0.94	From
2	2.90	1.48	67.0	0.70	12.0	2.68	2.73	red
3	4.18	2.13	87.0	1.17	9.6	1.00	1.15	heat
4	3.88	1.97	126.0	1.61	9.9	1.19	1.31	to
5	2.90	1.48	69.0	1.08	12.0	1.79	1.78	white
6	3.20	1.62	65.0	0.81	11.3	1.88	1.93	heat
					Mean	1.57	1.64	

Using the 1000 volts discharge potential.

$$v = \sqrt{\frac{2e}{m} V} = 1.77 \times 10^9 \ \frac{cm}{sec} .$$

CONCLUSIONS

The striking feature of the foregoing tables seem to be the fluctuating character of the values for $\frac{e}{m}$. The values for v we would of course expect to vary with different temperatures. However, it must be observed that although the individual values of $\frac{e}{m}$ might vary so much among themselves, still the average of any table will give a fair value of $\frac{e}{m}$. The varying nature of $\frac{e}{m}$ is however least for Table V, which is considered the best table of values by virtue of the favorable conditions under which the values were obtained, e.g., excellent, compact beams.

The values for $\frac{e}{m}$ in Table V vary from 1.27 to 2.07. A Wehnelt[38] seems to have had the same experience, for on inspection of his table of values for $\frac{e}{m}$ we find that his values varied from 1.34 to 1.81, giving an average value of 1.48. The mean value of $\frac{e}{m}$ in Table V is 1.61. Hence there seems to be fair agreement in that both values are somewhat lower than the best value of $\frac{e}{m}$ known, i.e., 1.76. In fact the tendency in the whole work was for $\frac{e}{m}$ to be less than the value just stated.

Considering v, we find again from Table V which was obtained under the most favorable conditions, that v varied from 1.00 to 1.75, while Wehnelt's values varied from 0.16 to 1.07 x 10^9. The difference is probably due to a difference in the temperatures employed in Wehnelt's and in this work. The order of v is however about the same. Knipp's values for v agree very well with those of this work, one value mentioned in his work being 1.6 x 10^9.

If we agree with J. J. Thomson[39] that electrons or corpuscles are

38 Ann. d. Phys., 14, p. 425, 1904.

39 Cond. of Elec. thru Gases, 2d. Ed., p. 197.

projected from incandescent substances, then it must be apparent that the emission of the electrons must be influenced by a great many causes. In the case of the Wehnelt cathode, the beam does not proceed from a point but from a small spot of lime, each particle of lime composing that spot not being perhaps at the same temperature, with the result that the beam is heterogeneous in nature. Then again the lime would continually be sputtering or falling off from the cathode, so that new lime had to be placed on the cathode after one hour's use. This gradual disappearance of the lime made it necessary to increase the heat of the Pt every now and then in order to keep the beam sharp and compact. We thus see that the source of the negative particles is not a steady one, and hence it should not be a matter of surprise if the values for $\frac{e}{m}$ obtained are found to vary among themselves. It might be said that the question of the efficiency of the hot lime cathode in the emission of negative particles, is not a settled one. In 1908 Frederick Soddy[40] found that the Wehnelt cathode lost its efficiency with time. On the other hand, R. S. Willows[41] and T. Picton found that the activity of the hot lime cathode increased with use. Recent work in this laboratory shows quite conclusively that the activity of the hot lime, where it is supplied by Bank of England sealing wax falls of rapidly with use, - the current rising to successively lower and lower maxima each succeeding day when heated to the same temperature.

In conclusion it might be said that although the Wehnelt cathode is not adapted for the most accurate determinations of $\frac{e}{m}$ for negative carriers of electricity, still the values obtained by the use of the

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40 Phys. Zeit., 9, No. 1, p. 8, 1908.

41 Proc. Phys. Soc. London, 23, p. 257, 1911.

helical method are confirmatory. The helical method is beautiful in nature and the apparatus serves very excellently as a demonstration piece for magnetic and electrostatic deflections.

Summary

A helical method for the determination of $\frac{e}{m}$ and v for the electrons from a Wehnelt cathode was devised.

The lime was supplied to the cathode by the application of a small quantity of Bank of England sealing wax.

The mean value of $\frac{e}{m}$ for a series in which the experimental conditions were best, was found to be 1.61×10^7. This agrees favorably with Wehnelt's value 1.48×10^7 determined by the usual method but is less than Classon's value of 1.77×10^7 determined by magnetic deflections (in a circular path) and potential differences of discharge method.

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